What can we learn from TMDs?

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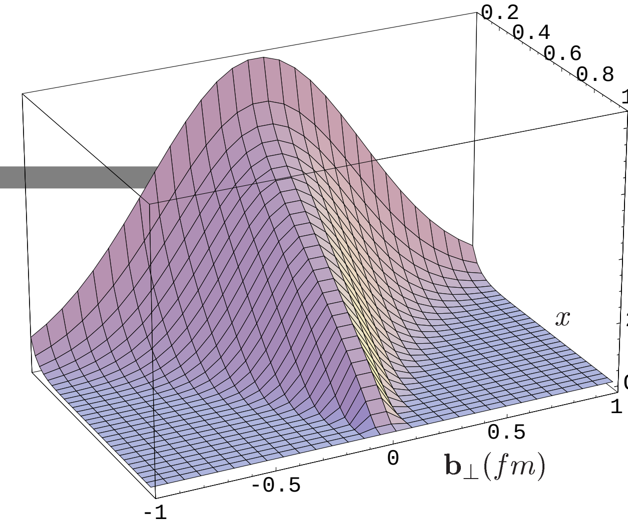
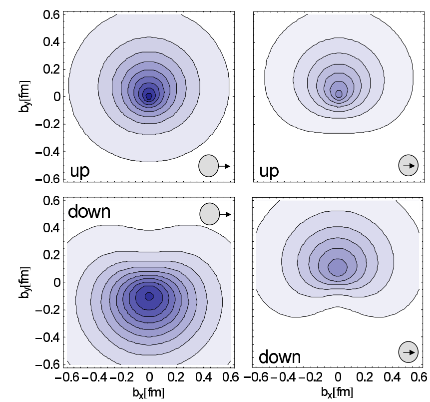
Outline

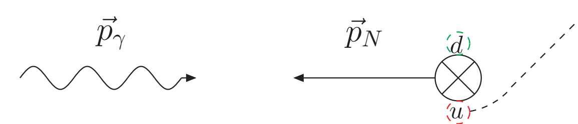


- Probabilistic interpretation of GPDs as Fourier transforms of impact parameter dependent PDFs
  - $H(x,0,-\mathbf{\Delta}_\perp^2) \longrightarrow q(x,\mathbf{b}_\perp)$
  - $\tilde{H}(x,0,-\mathbf{\Delta}_\perp^2) \longrightarrow \Delta q(x,\mathbf{b}_\perp)$
  - $E(x,0,-\mathbf{\Delta}_\perp^2) \longrightarrow \perp$ distortion of PDFs when the target is $\perp$ polarized



- Chromodynamik lensing and $\perp$ single-spin asymmetries (SSA)

$$\left.\begin{array}{l}\text{transverse distortion of PDFs}\\ \quad + \text{final state interactions}\end{array}\right\} \Rightarrow \quad \perp \text{SSA in} \quad \gamma N \longrightarrow \pi + X$$



  - Sivers
  - Boer-Mulders
  - peanuts, bagels, pretzels, worm-gear, ...

- Summary

Impact parameter dependent PDFs

- define $\perp$ localized state [D.Soper,PRD**15**, 1141 (1977)]

$$\left|p^+,\mathbf{R}_\perp=\mathbf{0}_\perp,\lambda\right\rangle \equiv \mathcal{N}\int d^2\mathbf{p}_\perp \left|p^+,\mathbf{p}_\perp,\lambda\right\rangle$$

  Note: $\perp$ boosts in IMF form Galilean subgroup $\Rightarrow$ this state has
  $\mathbf{R}_\perp \equiv \frac{1}{P^+}\int dx^- d^2\mathbf{x}_\perp\, \mathbf{x}_\perp T^{++}(x) = \sum_i x_i \mathbf{r}_{i,\perp} = \mathbf{0}_\perp$
  (cf.: working in CM frame in nonrel. physics)

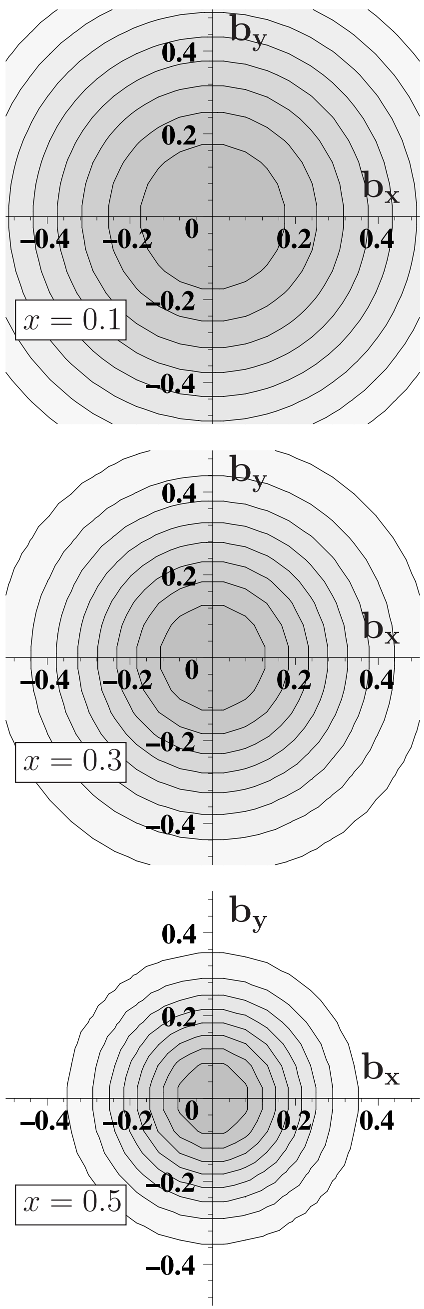
- define impact parameter dependent PDF

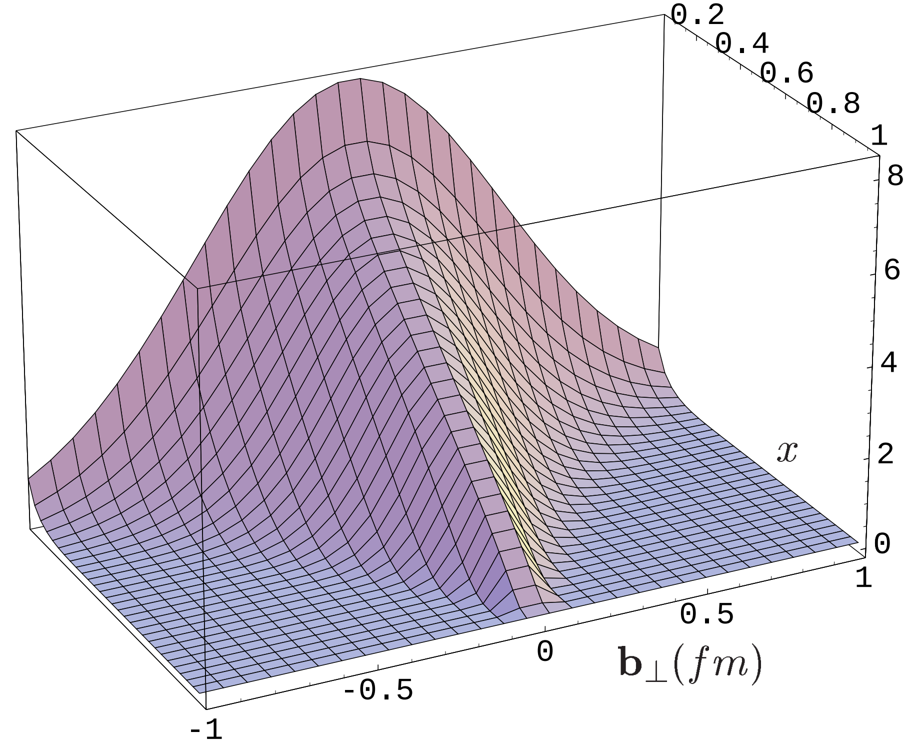
$$q(x,\mathbf{b}_\perp) \equiv \int \frac{dx^-}{4\pi}\left\langle p^+,\mathbf{R}_\perp=\mathbf{0}_\perp\right|\bar{q}(-\frac{x^-}{2},\mathbf{b}_\perp)\gamma^+ q(\frac{x^-}{2},\mathbf{b}_\perp)\left|p^+,\mathbf{R}_\perp=\mathbf{0}_\perp\right\rangle e^{ixp^+x^-}$$

$\hookrightarrow$

$$\begin{aligned} q(x,\mathbf{b}_\perp) &= \int \frac{d^2\mathbf{\Delta}_\perp}{(2\pi)^2} e^{i\mathbf{\Delta}_\perp\cdot\mathbf{b}_\perp} H(x,0,-\mathbf{\Delta}_\perp^2), \\ \Delta q(x,\mathbf{b}_\perp) &= \int \frac{d^2\mathbf{\Delta}_\perp}{(2\pi)^2} e^{i\mathbf{\Delta}_\perp\cdot\mathbf{b}_\perp} \tilde{H}(x,0,-\mathbf{\Delta}_\perp^2), \end{aligned}$$

$q(x, \mathbf{b}_\perp)$ for unpol. p





$x =$ momentum fraction of the quark

$\vec{b} = \perp$ position of the quark

Transversely Deformed Distributions and $E(x,0,-\mathbf{\Delta}_\perp^2)$

M.B., Int.J.Mod.Phys.A**18**, 173 (2003)

- So far: only unpolarized (or long. pol.) nucleon! In general ($\xi = 0$):

$$\int \frac{dx^-}{4\pi} e^{ip^+x^-x} \langle P{+}\Delta,\uparrow| \bar{q}(0)\, \gamma^+ q(x^-) | P,\uparrow\rangle = H(x,0,-\mathbf{\Delta}_\perp^2)$$

$$\int \frac{dx^-}{4\pi} e^{ip^+x^-x} \langle P{+}\Delta,\uparrow| \bar{q}(0)\, \gamma^+ q(x^-) | P,\downarrow\rangle = -\frac{\Delta_x - i\Delta_y}{2M} E(x,0,-\mathbf{\Delta}_\perp^2).$$

- Consider nucleon polarized in x direction (in IMF) $|X\rangle \equiv |p^+, \mathbf{R}_\perp = \mathbf{0}_\perp, \uparrow\rangle + |p^+, \mathbf{R}_\perp = \mathbf{0}_\perp, \downarrow\rangle$.

$\hookrightarrow$ unpolarized quark distribution for this state:

$$q(x,\mathbf{b}_\perp) = \mathcal{H}(x,\mathbf{b}_\perp) - \frac{1}{2M}\frac{\partial}{\partial b_y}\int \frac{d^2\mathbf{\Delta}_\perp}{(2\pi)^2} E(x,0,-\mathbf{\Delta}_\perp^2) e^{-i\mathbf{b}_\perp\cdot\mathbf{\Delta}_\perp}$$

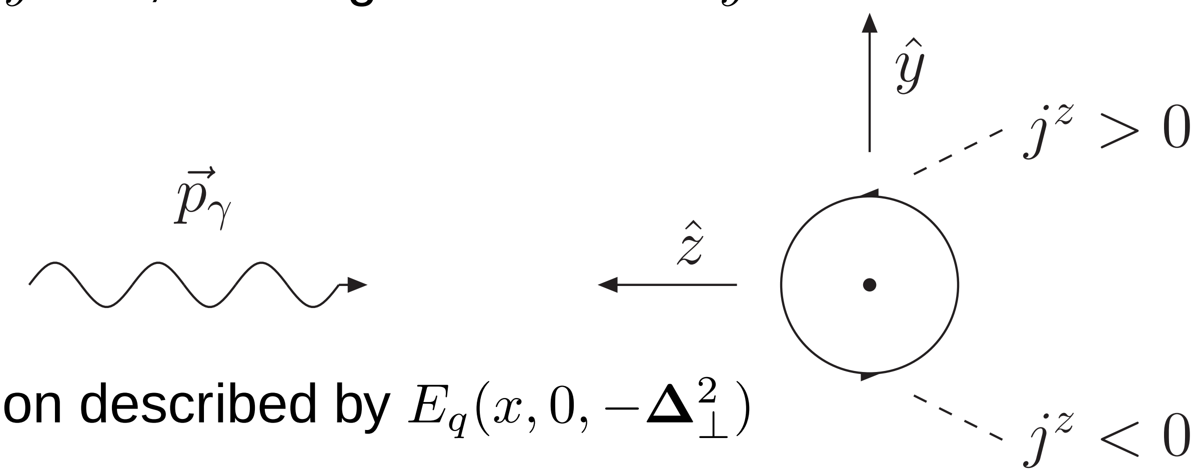
- Physics: $j^+ = j^0 + j^3$, and left-right asymmetry from j^3 ! [X.Ji, PRL **91**, 062001 (2003)]

Intuitive connection with $\vec{J}_q$

- DIS probes quark momentum density in the infinite momentum frame (IMF). Quark density in IMF corresponds to $j^+ = j^0 + j^3$ component in rest frame ($\vec{p}_{\gamma^*}$ in $-\hat{z}$ direction)

$\hookrightarrow$ j^+ larger than j^0 when quark current towards the γ^*; suppressed when away from γ^*

$\hookrightarrow$ For quarks with positive orbital angular momentum in $\hat{x}$-direction, j^z is positive on the $+\hat{y}$ side, and negative on the $-\hat{y}$ side



- Details of $\perp$ deformation described by $E_q(x, 0, -\mathbf{\Delta}_\perp^2)$

$\hookrightarrow$ not surprising that $E_q(x, 0, -\mathbf{\Delta}_\perp^2)$ enters Ji relation!

$$\langle J_q^i \rangle = S^i \int dx \left[H_q(x,0,0) + E_q(x,0,0) \right] x.$$

The Ji-relation (poor man's derivation)

- What distinguishes the Ji-decomposition from other decompositions is the fact that L_q can be constrained by experiment:

$$\langle \vec{J}_q \rangle = \vec{S} \int_{-1}^{1} dx\, x \left[H_q(x,\xi,0) + E_q(x,\xi,0) \right]$$

(nucleon at rest; $\vec{S}$ is nucleon spin)

$\hookrightarrow$ $L_q^z = J_q^z - \frac{1}{2}\Delta q$

- derivation (MB-version):
  - consider nucleon state that is an eigenstate under rotation about the $\hat{x}$-axis (e.g. nucleon polarized in $\hat{x}$ direction with $\vec{p} = 0$ (wave packet if necessary)
  - for such a state, $\langle T_q^{00} y \rangle = 0 = \langle T_q^{zz} y \rangle$ and $\langle T_q^{0y} z \rangle = -\langle T_q^{0z} y \rangle$

  $\hookrightarrow$ $\langle T_q^{++} y \rangle = \langle T_q^{0y} z - T_q^{0z} y \rangle = \langle J_q^x \rangle$

  $\hookrightarrow$ relate 2^{nd} moment of $\perp$ flavor dipole moment to J_q^x

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  $\hookrightarrow$ relate 2^{nd} moment of $\perp$ flavor dipole moment to J_q^x
  - effect sum of two effects:
    - $\langle T^{++} y\rangle$ for a point-like transversely polarized nucleon
    - $\langle T_q^{++} y\rangle$ for a quark relative to the center of momentum of a transversely polarized nucleon
  - 2^{nd} moment of $\perp$ flavor dipole moment for point-like nucleon

$$\psi = \begin{pmatrix} f(r) \\ \frac{\vec{\sigma}\cdot\vec{p}}{E+m} f(r) \end{pmatrix} \chi \qquad \text{with} \qquad \chi = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ 1 \end{pmatrix}$$

The Ji-relation (poor man's derivation)

- derivation (MB-version):
  - $T_q^{0z} = i\bar{q}\left(\gamma^0\partial^z + \gamma^z\partial^0\right)q$
  - since $\psi^\dagger\partial_z\psi$ is even under $y \rightarrow -y$, $i\bar{q}\gamma^0\partial^z q$ does not contribute to $\langle T^{0z}y\rangle$
  - $\hookrightarrow$ using $i\partial_0\psi = E\psi$, one finds

$$\begin{aligned}
\langle T^{0z}b_y\rangle &= E\int d^3r\psi^\dagger\gamma^0\gamma^z\psi y = E\int d^3r\psi^\dagger\begin{pmatrix} 0 & \sigma^z \\ \sigma^z & 0 \end{pmatrix}\psi y \\
&= \frac{2E}{E+M}\int d^3r\chi^\dagger\sigma^z\sigma^y\chi f(r)(-i)\partial^y f(r)y = \frac{E}{E+M}\int d^3r f^2(r)
\end{aligned}$$

  - consider nucleon state with $\vec{p}=0$, i.e. $E=M$ & $\int d^3r f^2(r) = 1$
  - $\hookrightarrow$ 2^{nd} moment of $\perp$ flavor dipole moment $\langle T_q^{++}y\rangle = \langle T^{0z}b_y\rangle = \frac{1}{2}$
  - $\hookrightarrow$ 'overall shift' of nucleon COM yields contribution $\frac{1}{2}\int dx\, xH_q(x,0,0)$ to $\langle T_q^{++}y\rangle$

The Ji-relation (poor man's derivation)

- spherically symmetric wave packet for Dirac particle with $J_x = \frac{1}{2}$ centered around the origin has $\perp$ center of momentum $\frac{1}{M}\langle T_q^{++} b_y \rangle$ not at origin, but at $\frac{1}{2M}$!
- consistent with

$$\frac{1}{2} = \langle J_x \rangle = \langle \left(T^{0z} b^y - T^{0y} b^z \right) \rangle = 2 \langle T^{0z} b^y \rangle = \langle T^{++} b^y \rangle$$

  - 'overall shift of $\perp$ COM yields contribution $\frac{1}{2}\int dx\, x H_q(x,0,0)$ to $\langle T_q^{++} b_y \rangle$
  - intrinsic distortion adds $\frac{1}{2}\int dx\, x E_q(x,0,0)$ to that

$\hookrightarrow$ Ji relation $\frac{1}{2} = \langle J_x \rangle = \frac{1}{2}\int dx\, x \left[H_q(x,0,0) + E_q(x,0,0) \right]$

Transversely Deformed PDFs and $E(x,0,-\mathbf{\Delta}_\perp^2)$

- $q(x,\mathbf{b}_\perp)$ in $\perp$ polarized nucleon is deformed compared to longitudinally polarized nucleons !
- mean $\perp$ deformation of flavor q ($\perp$ flavor dipole moment)

$$d_y^q \equiv \int dx \int d^2\mathbf{b}_\perp q_X(x,\mathbf{b}_\perp) b_y = \frac{1}{2M}\int dx E_q(x,0,0) = \frac{\kappa_q^p}{2M}$$

with $\kappa_{u/d}^p \equiv F_2^{u/d}(0) = \mathcal{O}(1-2) \quad \Rightarrow \quad d_y^q = \mathcal{O}(0.2fm)$

- simple model: for simplicity, make ansatz where $E_q \propto H_q$

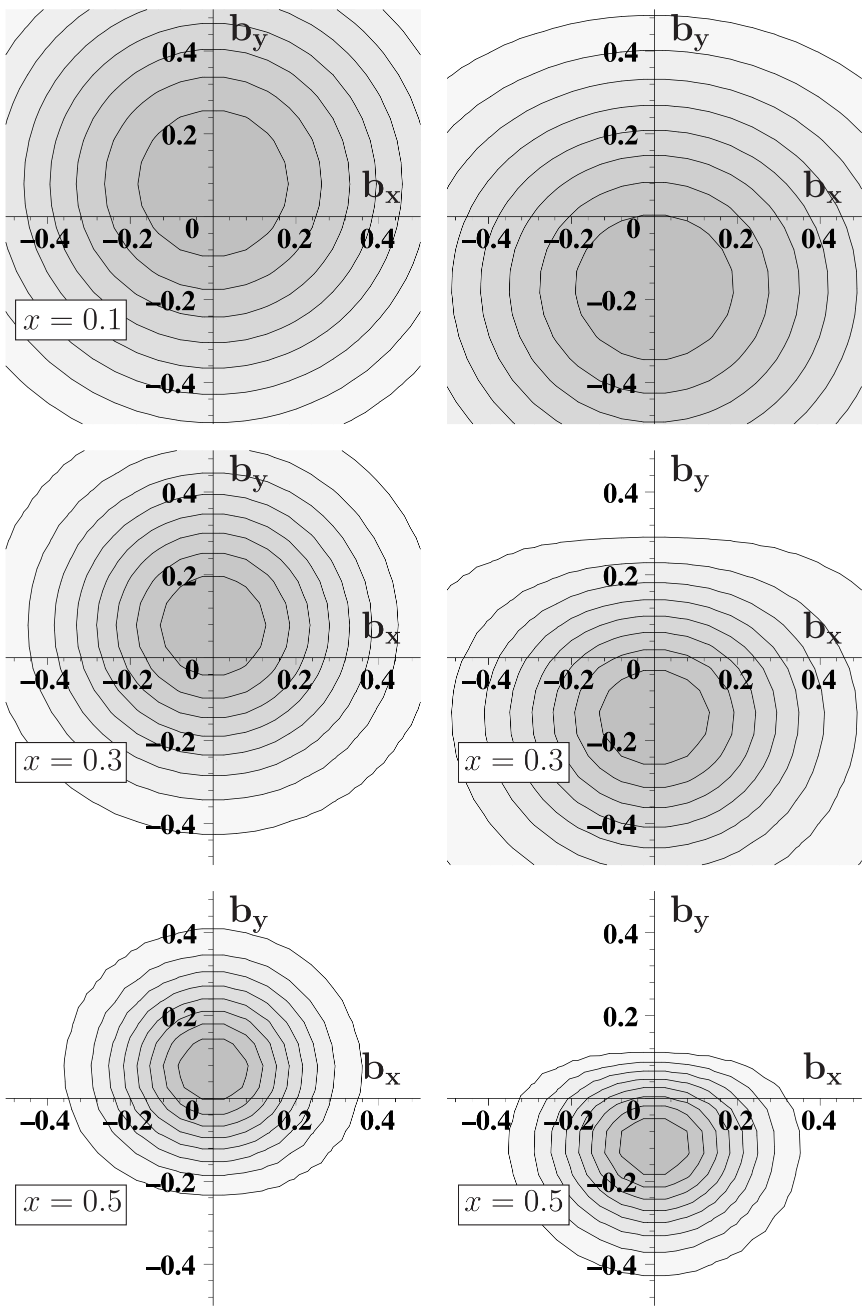
$$\begin{aligned} E_u(x,0,-\mathbf{\Delta}_\perp^2) &= \frac{\kappa_u^p}{2} H_u(x,0,-\mathbf{\Delta}_\perp^2) \\ E_d(x,0,-\mathbf{\Delta}_\perp^2) &= \kappa_d^p H_d(x,0,-\mathbf{\Delta}_\perp^2) \end{aligned}$$

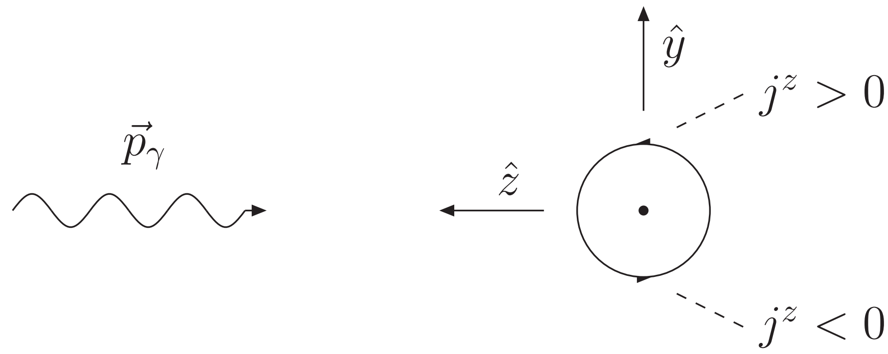
with $\kappa_u^p = 2\kappa_p + \kappa_n = 1.673 \qquad \kappa_d^p = 2\kappa_n + \kappa_p = -2.033.$

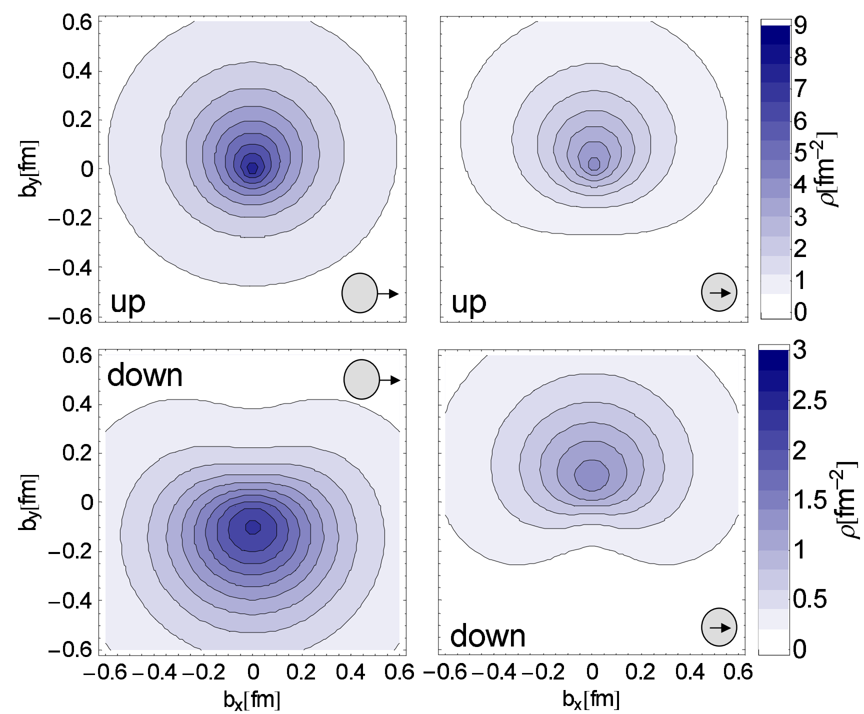
- Model too simple but illustrates that anticipated deformation is very significant since κ_u and κ_d known to be large!

p polarized in $+\hat{x}$ direction

$u(x, \mathbf{b}_\perp)$ $d(x, \mathbf{b}_\perp)$



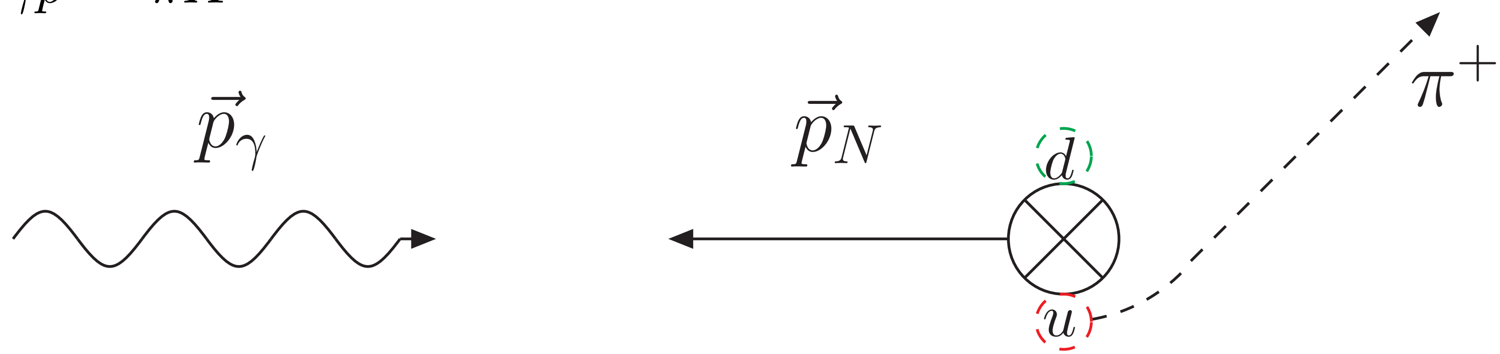




lattice (→ Ph.Hägler)

GPD $\longleftrightarrow$ SSA (Sivers)

- example: $\gamma p \to \pi X$



- u, d distributions in $\perp$ polarized proton have left-right asymmetry in $\perp$ position space (T-even!); sign "determined" by κ_u & κ_d
- attractive FSI deflects active quark towards the center of momentum

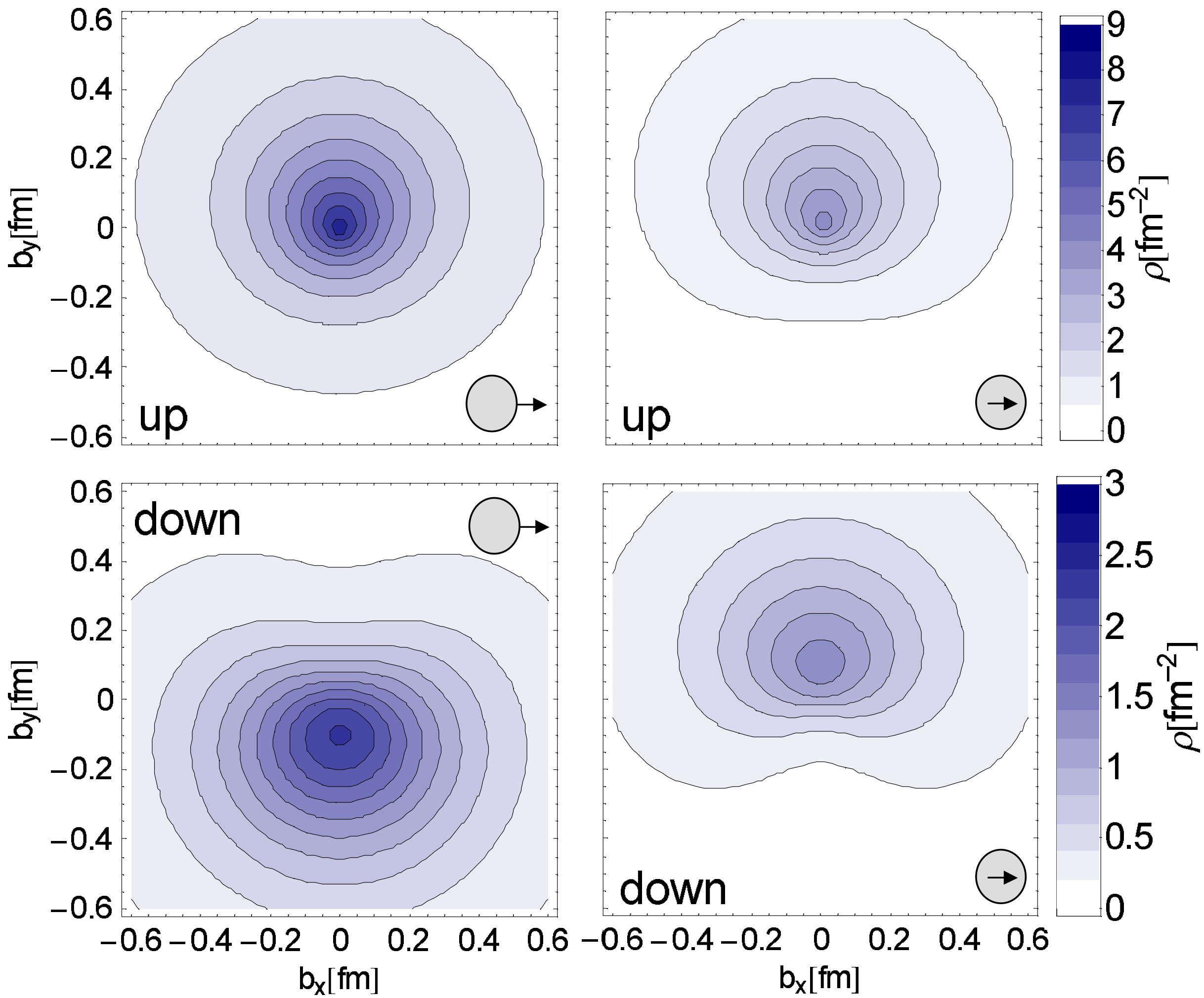
$\hookrightarrow$ FSI translates position space distortion (before the quark is knocked out) in $+\hat{y}$-direction into momentum asymmetry that favors $-\hat{y}$ direction

$\hookrightarrow$ correlation between sign of κ_q^p and sign of SSA: $f_{1T}^{\perp q} \sim -\kappa_q^p$

- $f_{1T}^{\perp q} \sim -\kappa_q^p$ confirmed by HERMES data (also consistent with COMPASS deuteron data $f_{1T}^{\perp u} + f_{1T}^{\perp d} \approx 0$)

IPDs on the lattice (→ Ph.Hägler)

- lowest moment of distribution $q(x, \mathbf{b}_\perp)$ for unpol. quarks in $\perp$ pol. proton (left) and of $\perp$ pol. quarks in unpol. proton (right):



Sivers $\overset{?}{\longleftrightarrow}$ GPDs

- intuitive picture ('chromodynamic lensing') sign of deformation $\leftrightarrow E(x,0,t) \leftrightarrow$ sign of Sivers
- quantitative relation: need model for FSI!
  - treat FSI to lowest order (implicit in many TMD models)
  - $\hookrightarrow$ average $\perp$ momentum of quarks with flavor q

$$\langle k_{i,q} \rangle = \frac{g}{4p^+} \int \frac{d^2\mathbf{y}_\perp}{2\pi} \frac{y_i}{\mathbf{y}_\perp^2} \langle P,S | \bar{q}(y)\gamma^+ \frac{\lambda^a}{2} q(y) \rho^a(\mathbf{0}_\perp) | P,S \rangle$$

  with $\rho^a(\mathbf{y}_\perp) = \int dy^- j^{+a}(y^-, \mathbf{y}_\perp)$
  - $\hookrightarrow$ sensitive to color density-density correlations
  - if quarks of flavor q are shifted to positive y_2 (e.g. u quarks in proton polarized in $+\hat{x}$ direction then y_2) then y_2 in interal more likely to be positive and,
  - $\hookrightarrow$ (incl. '-' from color wave function) $\langle k_{i,q} \rangle < 0$

Sivers $\overset{?}{\longleftrightarrow}$ GPDs

- valence wave function: color density-density correlation $\longrightarrow -\frac{4}{3}\times$ color-neutral density-density correlation

$$\langle k_{i,q}\rangle = \frac{g}{4p^+}\frac{4}{3}\int \frac{d^2\mathbf{y}_\perp}{2\pi}\frac{y_i}{\mathbf{y}_\perp^2}\langle P,S|\,\bar{q}(0)\gamma^+q(0)\rho(\mathbf{y}_\perp)\,|P,S\rangle$$

- more quantitative relations require further assumptions obout relation between single particle distribution in COM frame and density-density correlation (e.g. factorization)
- spectator models: 1-1 correspondence between single particle distribution in COM fram and density-density correlation (e.g. factorization) as impact parameter $\mathbf{b}_\perp$ (displacement from COM) and $\mathbf{r}_\perp$ (displacement from spectator) related $\mathbf{b}_\perp = (1-x)\mathbf{r}_\perp$

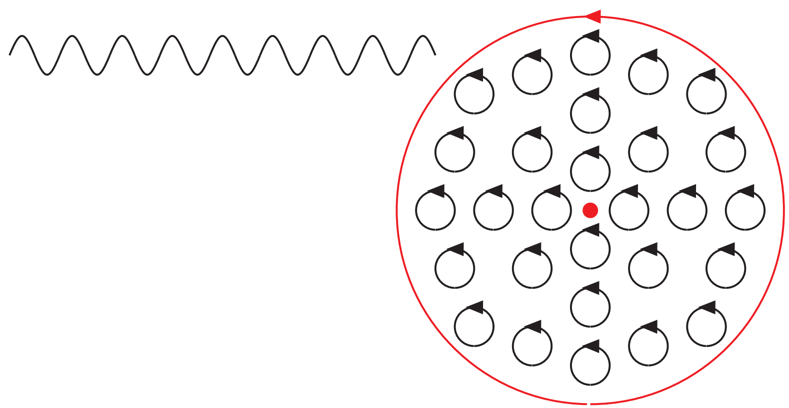
$\hookrightarrow$ chromodynamic lensing exact in such models

$\hookrightarrow$ possible to derive exact relations between Sivers and GPDs in such models ($\rightarrow$ I.Schmidt, A.Metz,...)

Transversity Distribution in Unpolarized Target (sign)

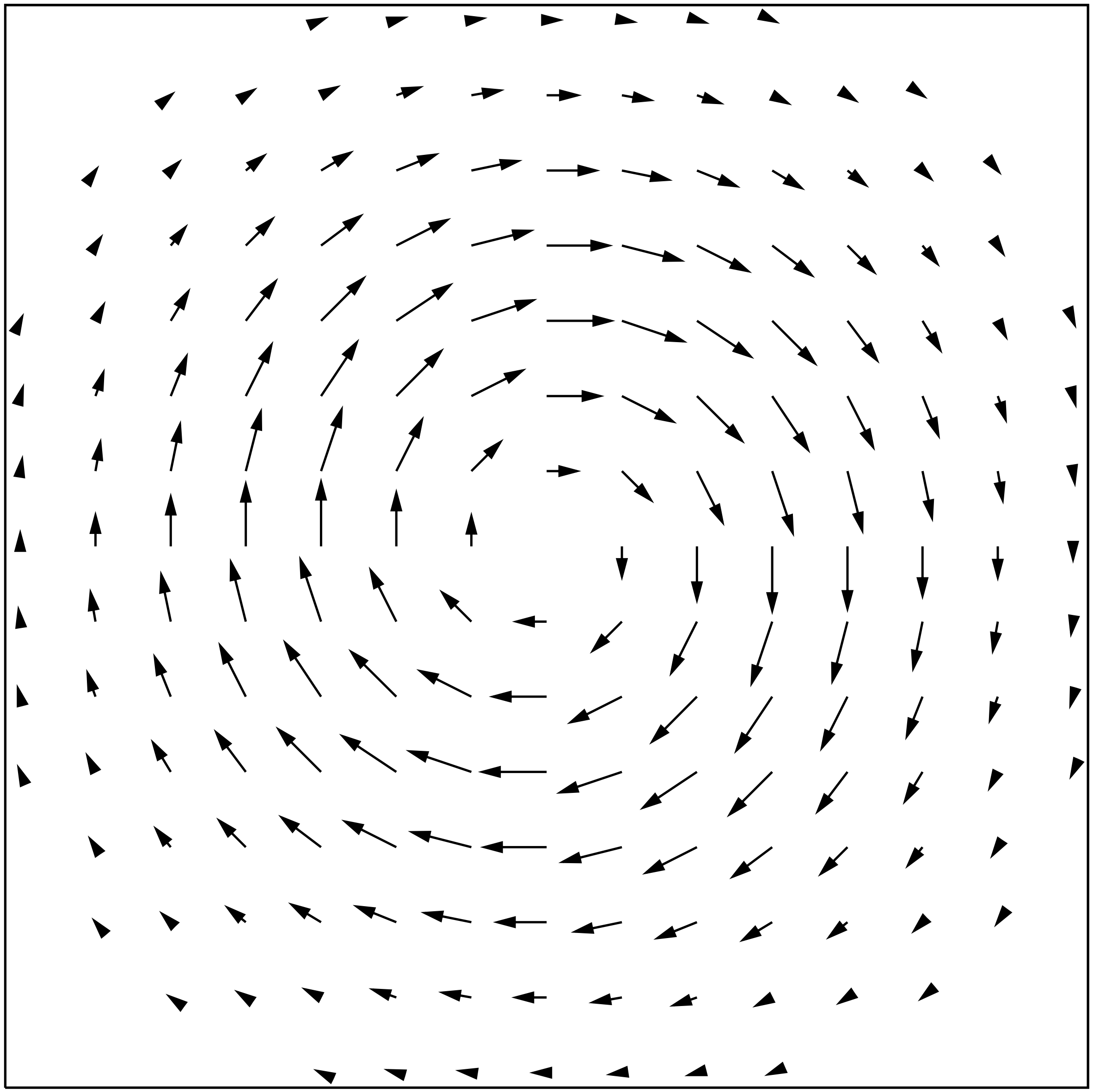
- Consider quark state with angular momentum out of the plane
  $\hookrightarrow$ that state has transversity out of plane
- expect counterclockwise net current $\bar{q}\vec{\gamma}q$ associated with the magnetization density in this state

$\hookrightarrow$ $\bar{q}\gamma^z q$ pos. at the top and neg. at bottom



- virtual photon 'sees' enhancement of quarks with transversity out of plane at the top, and transversity into plane at bottom
- physics: sideways shift of COM: $\langle J_q^y \rangle \leftrightarrow \int dz^- d^2\mathbf{z}_\perp \langle T^{++}(z) z^y \rangle$

Transversity Distribution in Unpolarized Target



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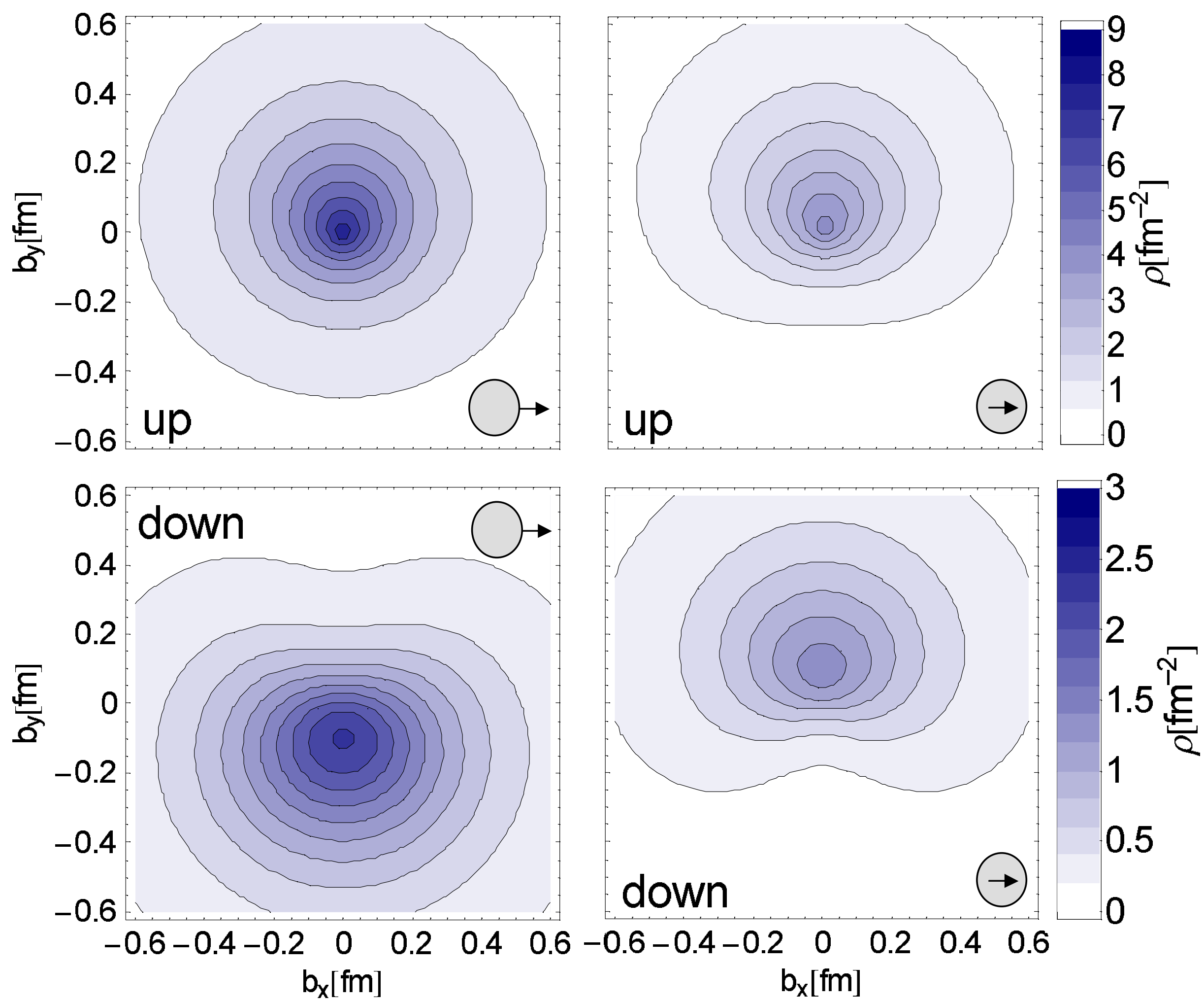


Boer-Mulders Function

- SIDIS: attractive FSI expected to convert position space asymmetry into momentum space asymmetry

$\hookrightarrow$ e.g. quarks at negative b_x with spin in $+\hat{y}$ get deflected (due to FSI) into $+\hat{x}$ direction

$\hookrightarrow$ Interpretation of $M^2 d_2 \equiv 3M^2 \int dx x^2 \bar{g}_2(x)$ as $\perp$ **force on active quark in DIS in the instant after being struck by the virtual photon**

$$\langle F^y(0) \rangle = -M^2 d_2 \qquad \text{(rest frame; } S^x = 1\text{)}$$

- In combination with measurements of f_2
  - color-electric/magnetic force $\frac{M^2}{4}\chi_E$ and $\frac{M^2}{2}\chi_M$
- $\kappa^{q/p} \Rightarrow \perp$ deformation $\Rightarrow d_2^{u/p} > 0$ & $d_2^{d/p} < 0$ (attractive FSI)
- combine measurement of d_2 with that of $f_{1T}^{\perp} \Rightarrow$ **range of FSI**
- x^2-moment of chirally odd twist-3 PDF $e(x) \longrightarrow$ **transverse force on transversly polarized quark in unpolarized target** ($\leftrightarrow$ Boer-Mulders $h_1^{\perp}$)(qualitative) connection between Boer-Mulders function $h_1^{\perp}(x, \mathbf{k}_{\perp})$ and the chirally odd GPD $\bar{E}_T$ that is similar to

probing BM function in tagged SIDIS

- how do you measure the transversity distribution of quarks without measuring the transversity of a quark?
- consider semi-inclusive pion production off unpolarized target
- spin-orbit correlations in target wave function provide correlation between (primordial) quark transversity and impact parameter

$\hookrightarrow$ (attractive) FSI provides correlation between quark spin and $\perp$ quark momentum $\Rightarrow$ BM function

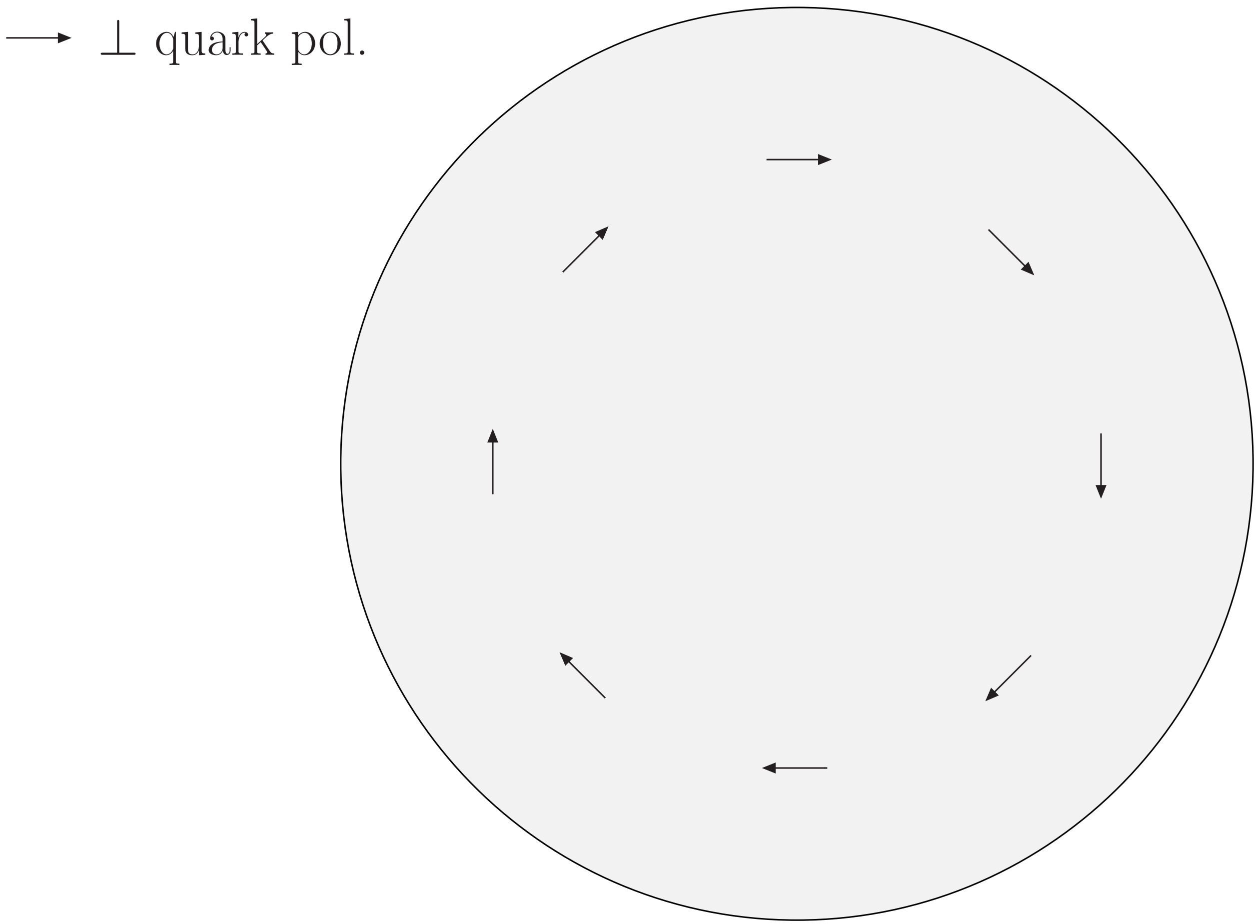
- Collins effect: left-right asymmetry of π distribution in fragmentation of $\perp$ polarized quark $\Rightarrow$ 'tag' quark spin

$\hookrightarrow$ $\cos(2\phi)$ modulation of π distribution relative to lepton scattering plane

$\hookrightarrow$ $\cos(2\phi)$ asymmetry proportional to: Collins $\times$ BM

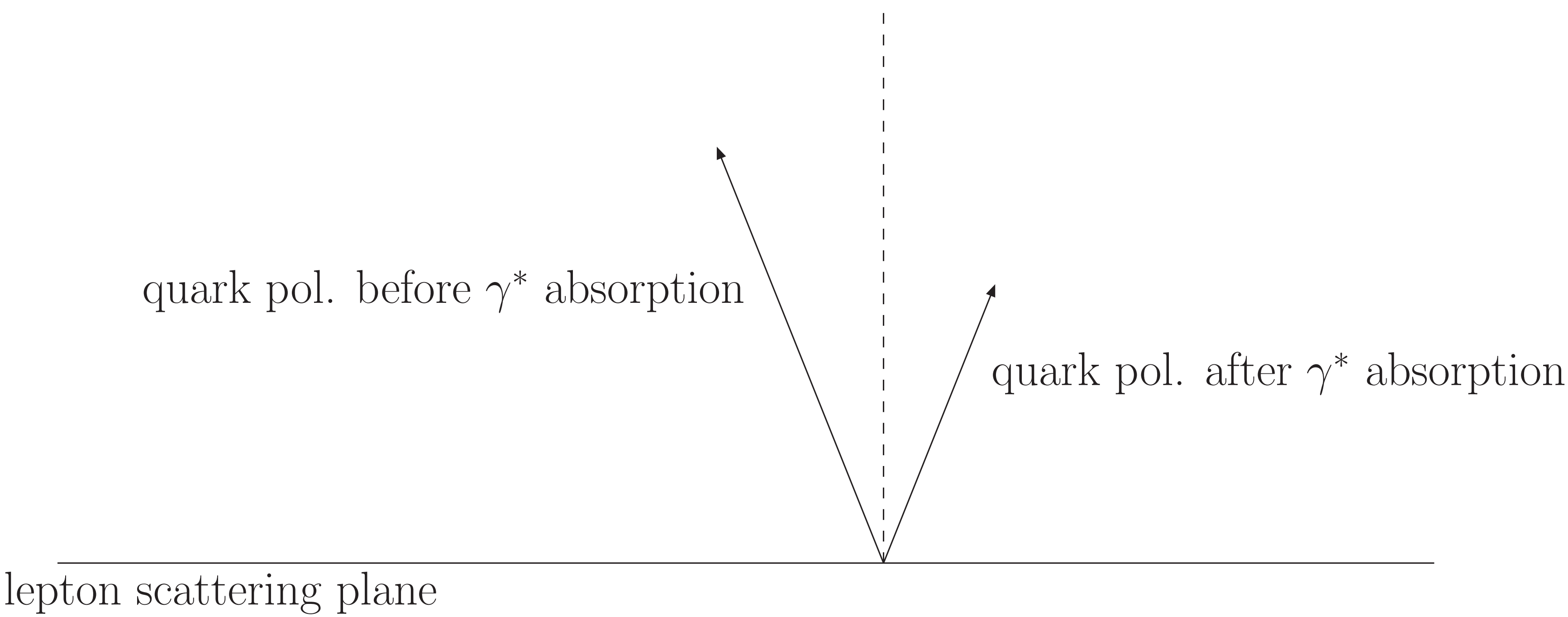
probing BM function in tagged SIDIS

Primordial Quark Transversity Distribution



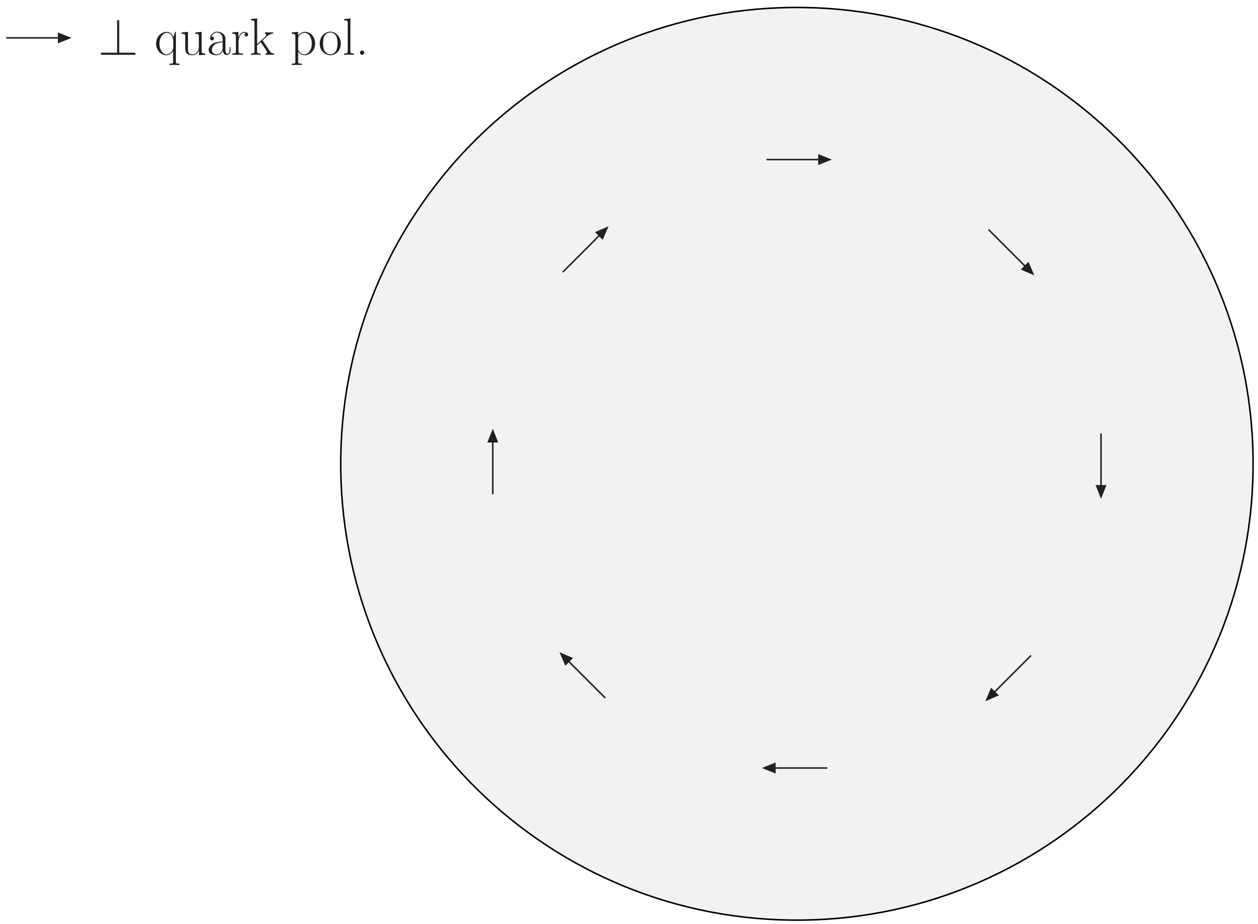
$\perp$ polarization and γ^* absorption

- QED: when the γ^* scatters off $\perp$ polarized quark, the $\perp$ polarization gets modified
  - gets reduced in size
  - gets tilted symmetrically w.r.t. normal of the scattering plane



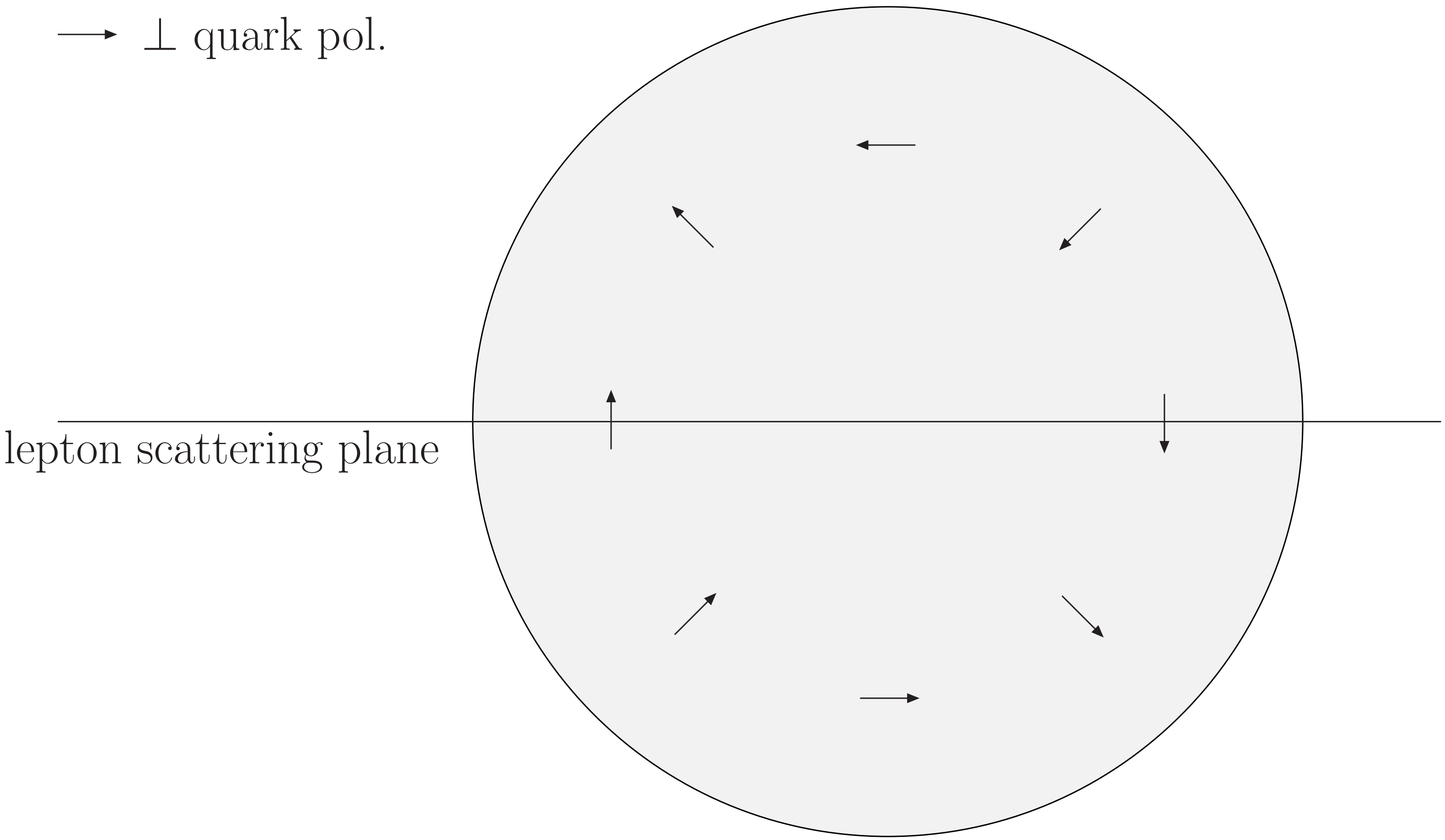
probing BM function in tagged SIDIS

Primordial Quark Transversity Distribution



probing BM function in tagged SIDIS

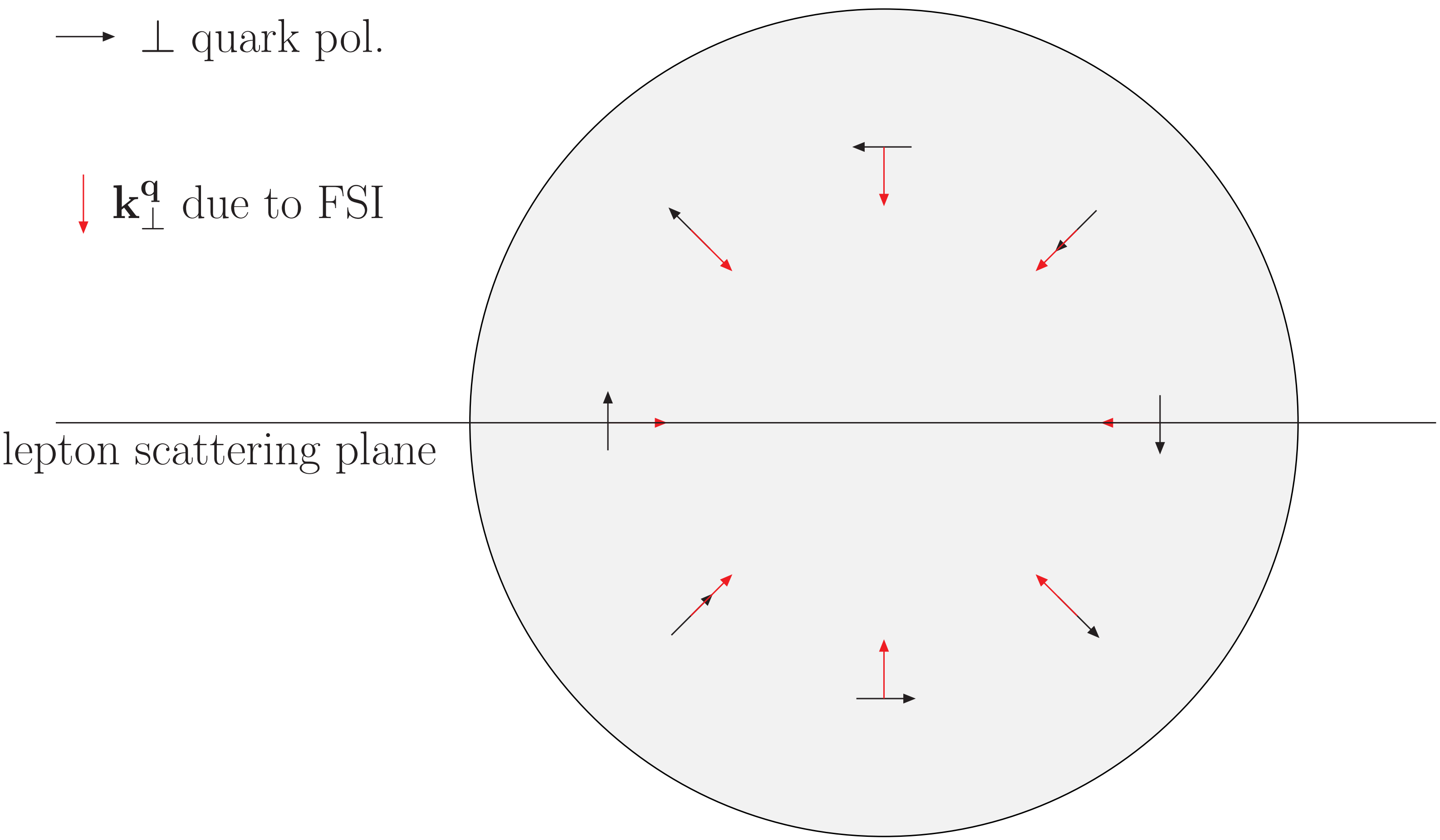
Quark Transversity Distribution after γ^* absorption



quark transversity component in lepton scattering plane flips

probing BM function in tagged SIDIS

$\perp$ momentum due to FSI

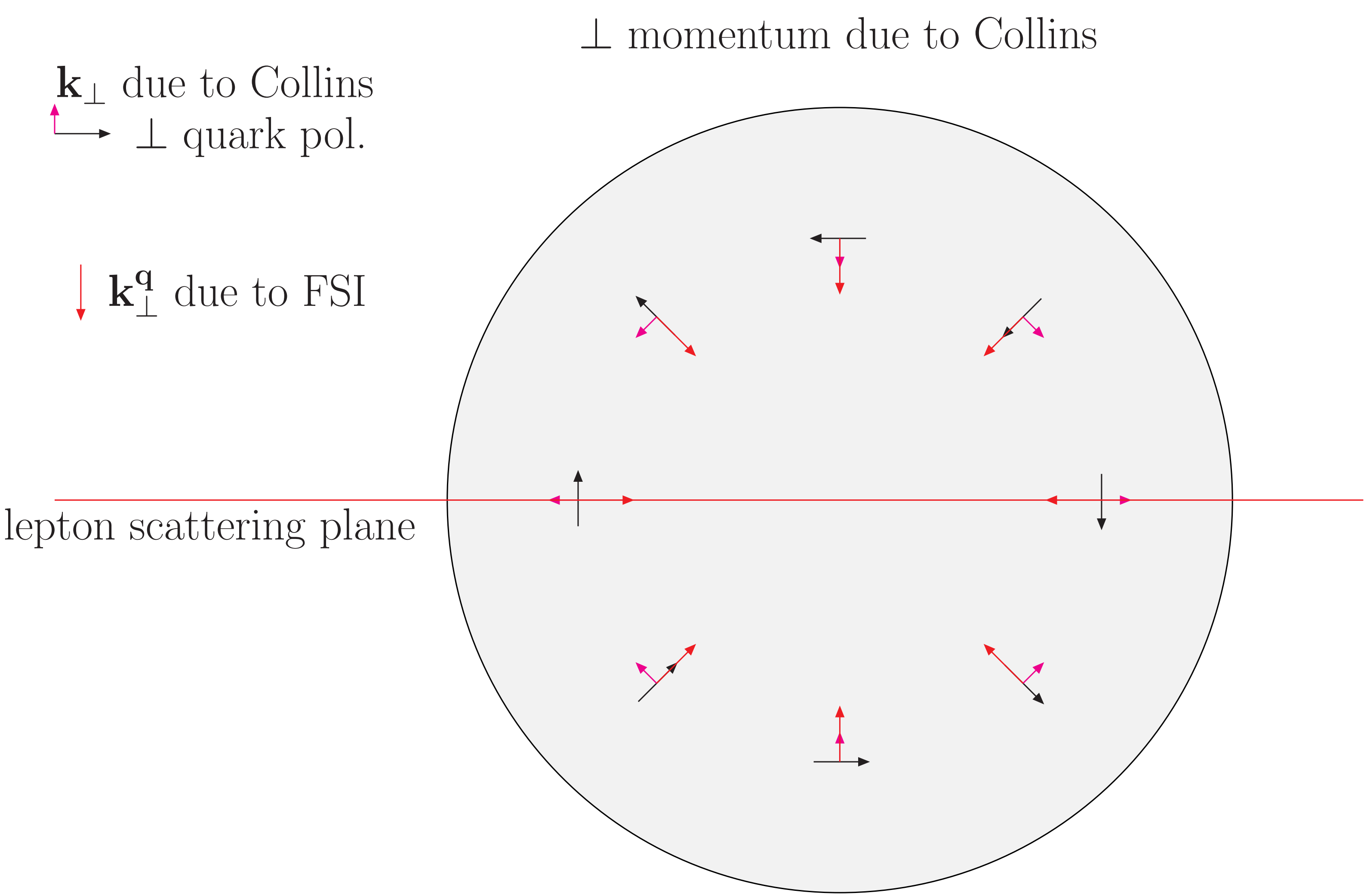


on average, FSI deflects quarks towards the center

Collins effect

- When a $\perp$ polarized struck quark fragments, the strucure of jet is sensitive to polarization of quark
- distribution of hadrons relative to $\perp$ polarization direction may be left-right asymmetric
- asymmetry parameterized by **Collins fragmentation function**
- Artru model:
  - struck quark forms pion with $\bar{q}$ from $q\bar{q}$ pair with ${}^3P_0$ 'vacuum' quantum numbers
  - $\hookrightarrow$ pion 'inherits' OAM in direction of $\perp$ spin of struck quark
  - $\hookrightarrow$ produced pion preferentially moves to left when looking into direction of motion of fragmenting quark with spin up
- Artru model confirmed by HERMES experiment
- more precise determination of Collins function under way (KEK)

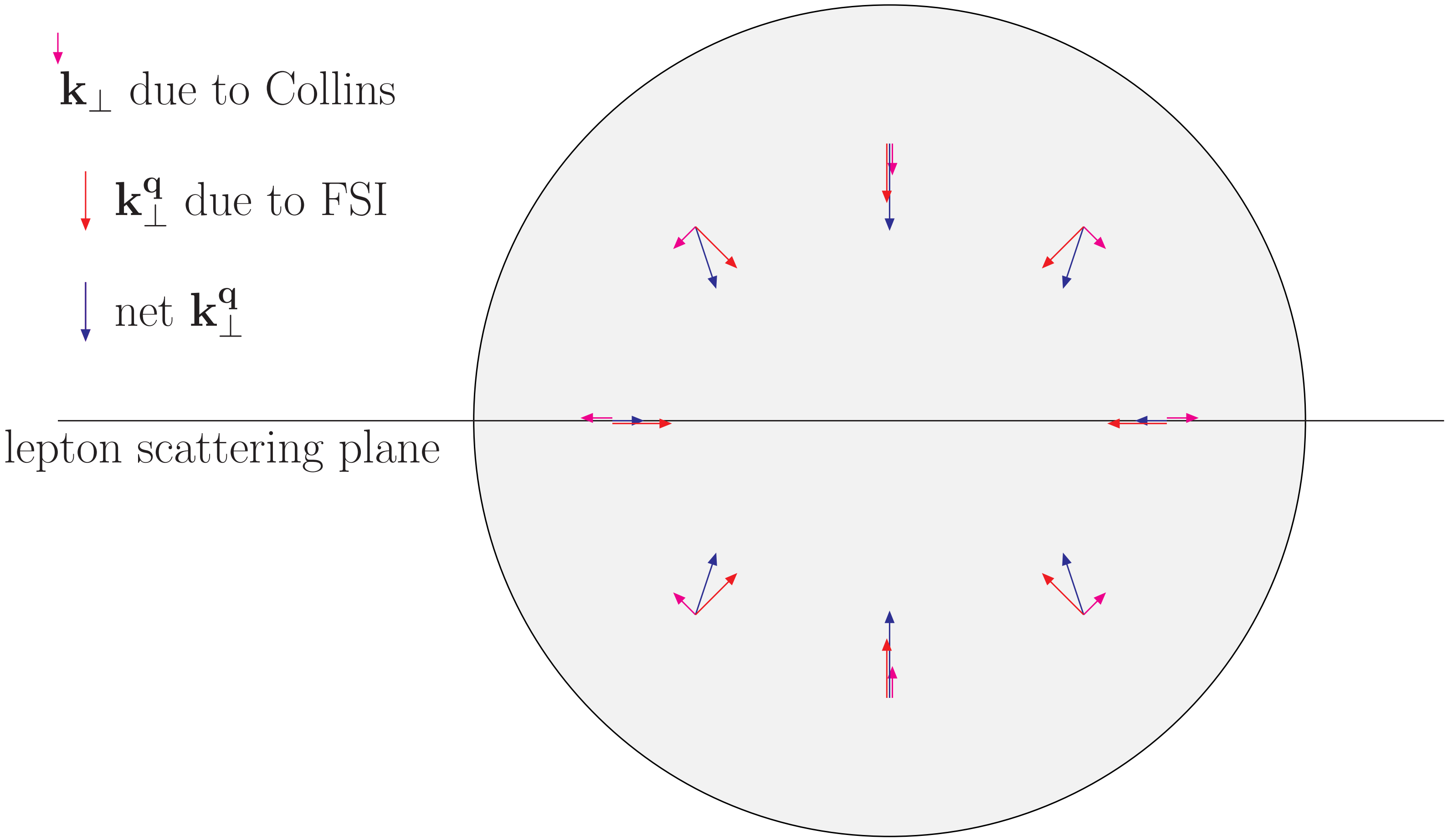
probing BM function in tagged SIDIS



SSA of π in jet emanating from $\perp$ pol. q

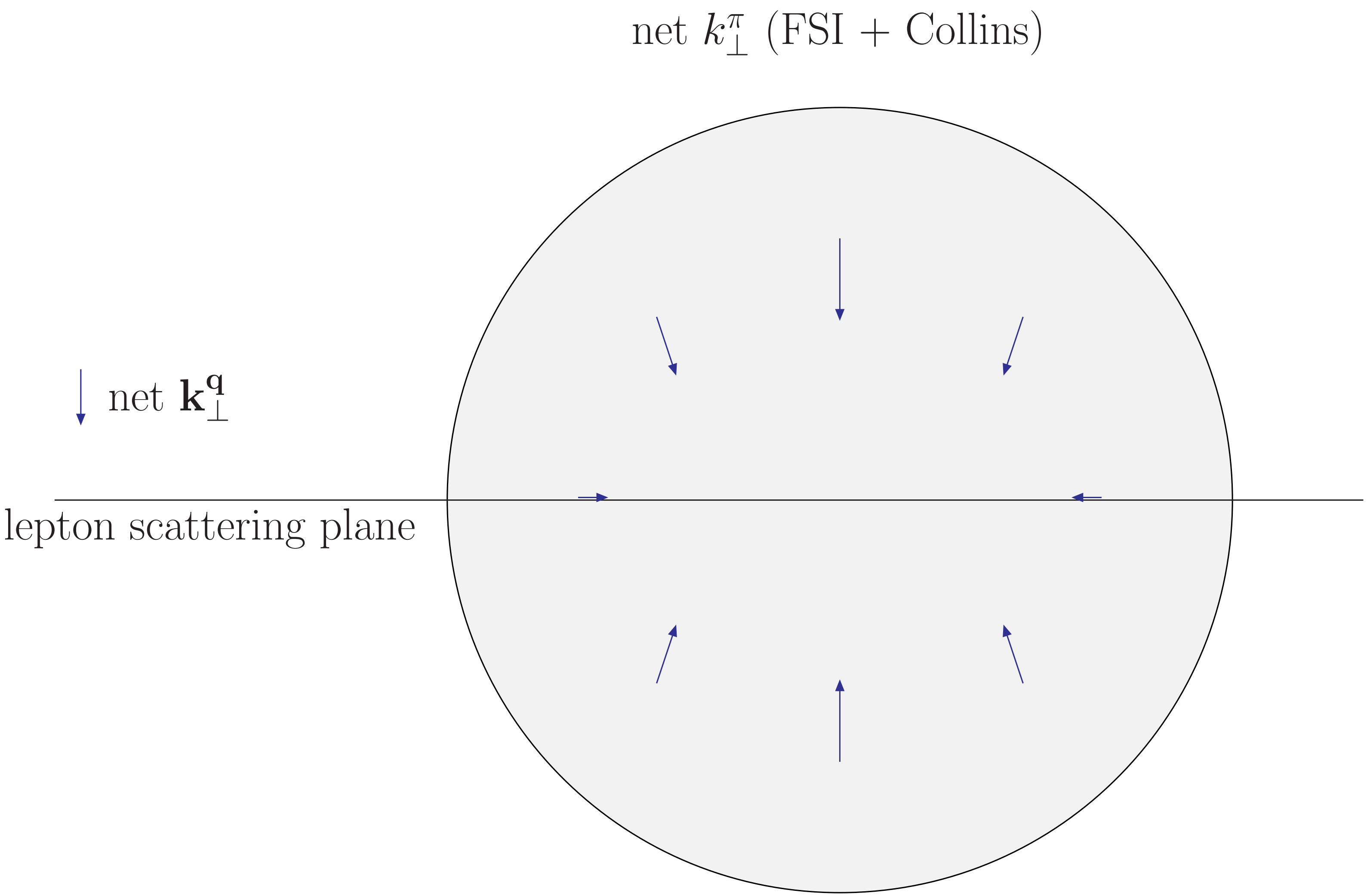
probing BM function in tagged SIDIS

net $\perp$ momentum (FSI+Collins)



$\hookrightarrow$ in this example, enhancement of pions with $\perp$ momenta $\perp$ to lepton plane

probing BM function in tagged SIDIS



$\hookrightarrow$ expect enhancement of pions with $\perp$ momenta $\perp$ to lepton plane

π^+/π^- $\cos 2\phi$ **asymmetry**

- including both favored $H_{1f}^\perp$ and unfavored $H_{1u}^\perp$ fragmentation one finds for the contribution from Boer-Mulders-Collins to the $\cos 2\phi$ moment of the X-section

$$\sigma_{\pi^+}^{\cos 2\phi} = h_{1u}^\perp \times H_{1,fav}^\perp + h_{1d}^\perp \times H_{1,unfav}^\perp$$

$$\sigma_{\pi^-}^{\cos 2\phi} = h_{1d}^\perp \times H_{1,fav}^\perp + h_{1u}^\perp \times H_{1,unfav}^\perp$$

- useful linear combinations

$$\sigma_{\pi^+}^{\cos 2\phi} - \sigma_{\pi^-}^{\cos 2\phi} = (h_{1u}^\perp - h_{1d}^\perp) \times (H_{1,fav}^\perp - H_{1,unfav}^\perp)$$

$$\sigma_{\pi^+}^{\cos 2\phi} + \sigma_{\pi^-}^{\cos 2\phi} = (h_{1u}^\perp + h_{1d}^\perp) \times (H_{1,fav}^\perp + H_{1,unfav}^\perp)$$

$h_{1T}^{\perp}$

- multiplies $s^i(2k^ik^j - \mathbf{k}_\perp^2\delta^{ij}S^j$, where s^i quark transversity, and S^j nucleon transverse spin
- for example, $h_{1T}^{\perp} > 0$ implies nucleon prolate when quark transversity parallel nucleon spin

- and more oblate when quark transversity anti-parallel nucleon spin

- and for some spin configurations may even resemble a pretzel ... (G.A. Miller, 2003)

$h_{1T}^{\perp}$

- contributes to matrix elements where both quark- and nucleon helicities flip — in opposite directions

$\hookrightarrow$ may change quark OAM by two units (p-p or s-d interference)

- p-p: consider quark target with $j_x = \frac{1}{2}$
  - upper Dirac component spherically symmetric (s-wave), but
  - lower component (p-wave) has either quark spin parallel j_x, and $l = 1, l_x = 0$ (prolate) or quark spin anti-parallel j_x and $l = 1, l_x = +1$ (oblate)
  - note: transversity $\neq$ transverse spin! Different sign for lower component...

  $\hookrightarrow$ oblate when quark transversity parallel j_x and prolate when quark transversity anti-parallel j_x

  $\hookrightarrow$ $h_{1T}^{\perp} < 0$

  - suggests $h_{1T}^{\perp,u} < 0$ and $h_{1T}^{\perp,d} > 0$ (consistent with lattice ($\longrightarrow$ Ph.Hägler) and models ($\longrightarrow$ M.Radici; S.Boffi; ...)

g_{1T} and $h_{1L}^{\perp}$

- g_{1T} multiplies $\lambda S^i k^i$ in TMD (λ = quark helicity):

$\hookrightarrow$ distribution of longitudinally polarized quarks in $\perp$ polarized nucleon!

- $h_{1L}^{\perp}$ multiplies $\Lambda s^i k^i$ (Λ = nucleon long. pol.)

$\hookrightarrow$ distribution of quark transversity in longitudinally polarized nucleon!

- in 'rest frame' (i.e. with $\gamma^+ \rightarrow \gamma^0$), both would vanish by rotational invariance
- can be generated by a boost to the IMF 'Melosh rotation', e.g. quarks with $\perp$ momentum and polarization acquire long. polarization component after boost to IMF (compare Thomas precession)

Summary

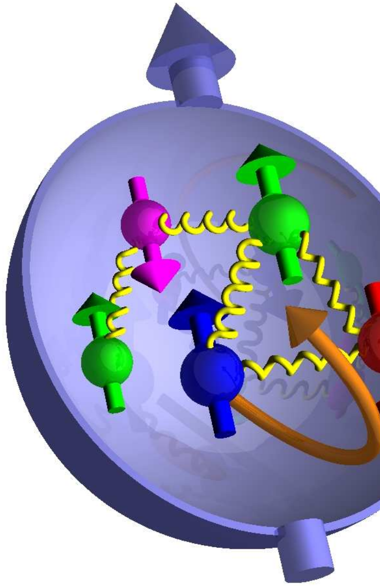
- GPDs $\stackrel{FT}{\longleftrightarrow}$ IPDs (impact parameter dependent PDFs)
- $E(x,0,-\mathbf{\Delta}_\perp^2) \longrightarrow \perp$ deformation of PDFs for $\perp$ polarized target

$\hookrightarrow$ $\kappa^{q/p}$ $\Rightarrow$ sign of deformation

$\hookrightarrow$ attractive FSI $\Rightarrow f_{1T}^{\perp u} < 0$ & $f_{1T}^{\perp d} > 0$

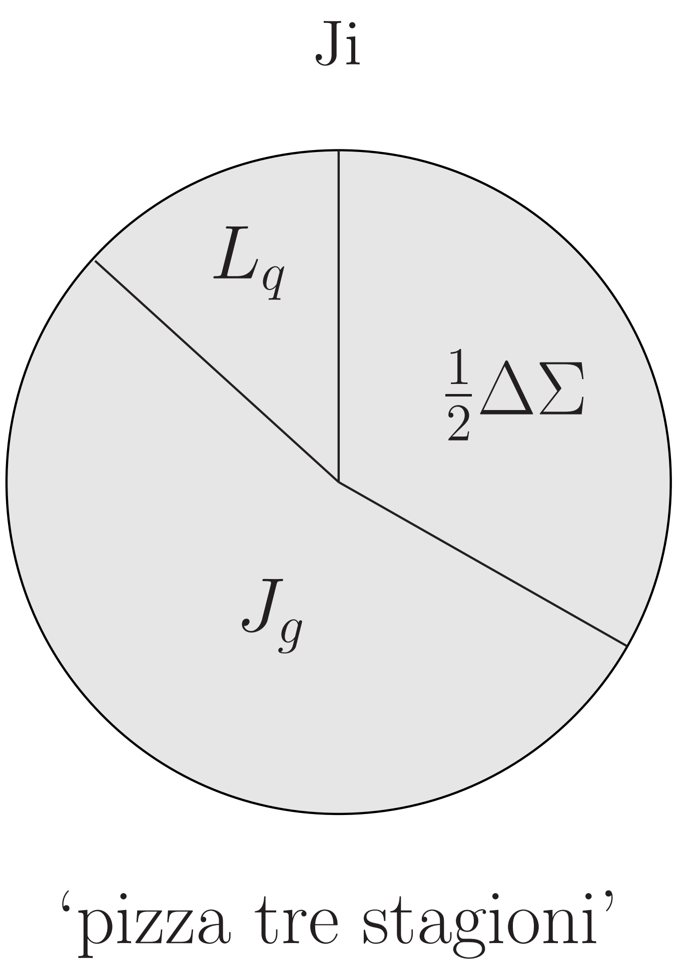
- 'parton interpretation' of Ji relation in terms as 'transverse shift' of T^{++}
- $\bar{E}_T \leftrightarrow -h_1^\perp$
- peanuts, donuts, pretzels, worm-gears

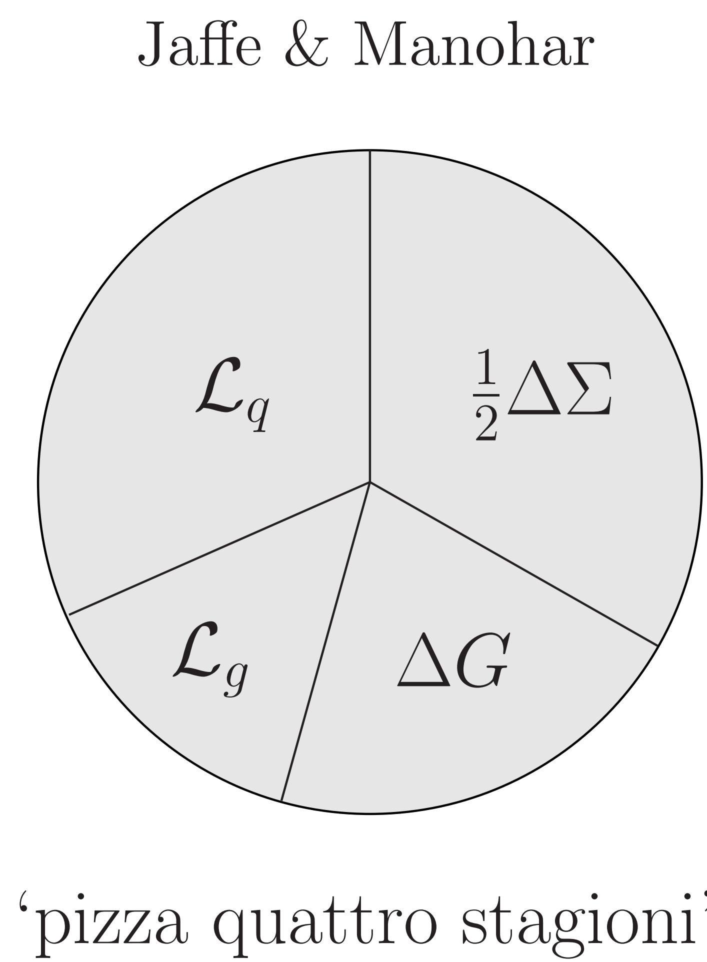
What is Orbital Angular Momentum?



- Ji decomposition
- Jaffe decomposition
- recent lattice results (Ji decomposition)
- model/QED illustrations for Ji v. Jaffe

The nucleon spin pizza(s)





- only $\frac{1}{2}\Delta\Sigma \equiv \frac{1}{2}\sum_q \Delta q$ common to both decompositions!

Angular Momentum Operator

- angular momentum tensor $M^{\mu\nu\rho} = x^\mu T^{\nu\rho} - x^\nu T^{\mu\rho}$
- $\partial_\rho M^{\mu\nu\rho} = 0$

$\hookrightarrow$ $\tilde{J}^i = \frac{1}{2}\varepsilon^{ijk} \int d^3r M^{jk0}$ conserved

$$\frac{d}{dt}\tilde{J}^i = \frac{1}{2}\varepsilon^{ijk} \int d^3x \partial_0 M^{jk0} = \frac{1}{2}\varepsilon^{ijk} \int d^3x \partial_l M^{jkl} = 0$$

- $M^{\mu\nu\rho}$ contains time derivatives (since $T^{\mu\nu}$ does)
  - use eq. of motion to get rid of these (as in T^{0i})
  - integrate total derivatives appearing in T^{0i} by parts
  - yields terms where derivative acts on x^i which then 'disappears'

  $\hookrightarrow$ J^i usally contains both
    - 'Extrinsic' terms, which have the structure '$\vec{x}\times$ Operator', and can be identified with 'OAM'
    - 'Intrinsic' terms, where the factor $\vec{x}\times$ does not appear, and can be identified with 'spin'

Angular Momentum in QCD (Ji)

- following this general procedure, one finds in QCD

$$\vec{J} = \int d^3x \left[\psi^\dagger \vec{\Sigma} \psi + \psi^\dagger \vec{x} \times \left(i\vec{\partial} - g\vec{A} \right) \psi + \vec{x} \times \left(\vec{E} \times \vec{B} \right) \right]$$

  with $\Sigma^i = \frac{i}{2} \varepsilon^{ijk} \gamma^j \gamma^k$

- Ji does <u>not</u> integrate gluon term by parts, <u>nor</u> identify gluon spin/OAM separately
- Ji-decomposition valid for all three components of $\vec{J}$, but usually only applied to $\hat{z}$ component, where the quark spin term has a partonic interpretation

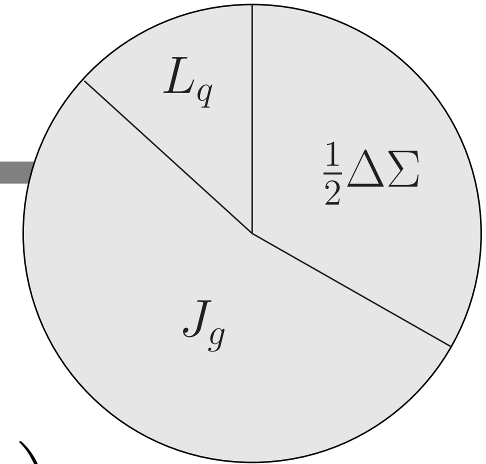
(+) all three terms manifestly gauge invariant

(+) DVCS can be used to probe $\vec{J}_q = \vec{S}_q + \vec{L}_q$

(-) quark OAM contains interactions

(-) only quark spin has partonic interpretation as a single particle density

Ji-decomposition



- Ji (1997)

$$\frac{1}{2} = \sum_q J_q + J_g = \sum_q \left(\frac{1}{2}\Delta q + L_q\right) + J_g$$

with ($P^\mu = (M,0,0,1)$, $S^\mu = (0,0,0,1)$)

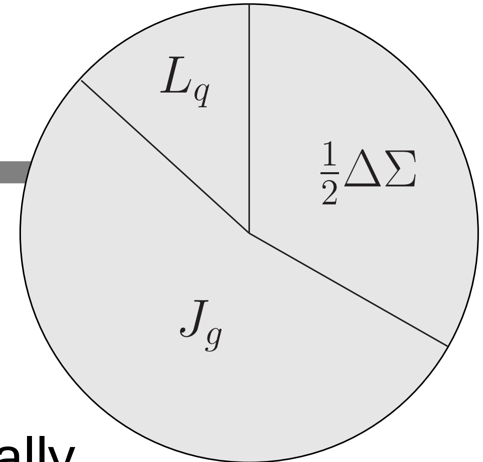
$$\frac{1}{2}\Delta q = \frac{1}{2}\int d^3x \, \langle P,S|\, q^\dagger(\vec{x})\Sigma^3 q(\vec{x}) \,|P,S\rangle \qquad \Sigma^3 = i\gamma^1\gamma^2$$

$$L_q = \int d^3x \, \langle P,S|\, q^\dagger(\vec{x}) \left(\vec{x}\times i\vec{D}\right)^3 q(\vec{x}) \,|P,S\rangle$$

$$J_g = \int d^3x \, \langle P,S| \left[\vec{x}\times\left(\vec{E}\times\vec{B}\right)\right]^3 |P,S\rangle$$

- $i\vec{D} = i\vec{\partial} - g\vec{A}$

Ji-decomposition



- $\vec{J} = \sum_q \frac{1}{2} q^\dagger \vec{\Sigma} q + q^\dagger \left(\vec{r} \times i\vec{D} \right) q + \vec{r} \times \left(\vec{E} \times \vec{B} \right)$
  applies to each vector component of nucleon angular momentum, but Ji-decomposition usually applied only to $\hat{z}$ component where at least quark spin has parton interpretation as difference between number densities
- Δq from polarized DIS
- $J_q = \frac{1}{2}\Delta q + L_q$ from exp/lattice (GPDs)
- L_q in principle independently defined as matrix elements of $q^\dagger \left(\vec{r} \times i\vec{D} \right) q$, but in practice easier by subtraction $L_q = J_q - \frac{1}{2}\Delta q$
- J_g in principle accessible through gluon GPDs, but in practice easier by subtraction $J_g = \frac{1}{2} - J_q$
- further decomposition of J_g into intrinsic (spin) and extrinsic (OAM) that is local and manifestly gauge invariant has not been found

L_q for proton from Ji-relation (lattice)

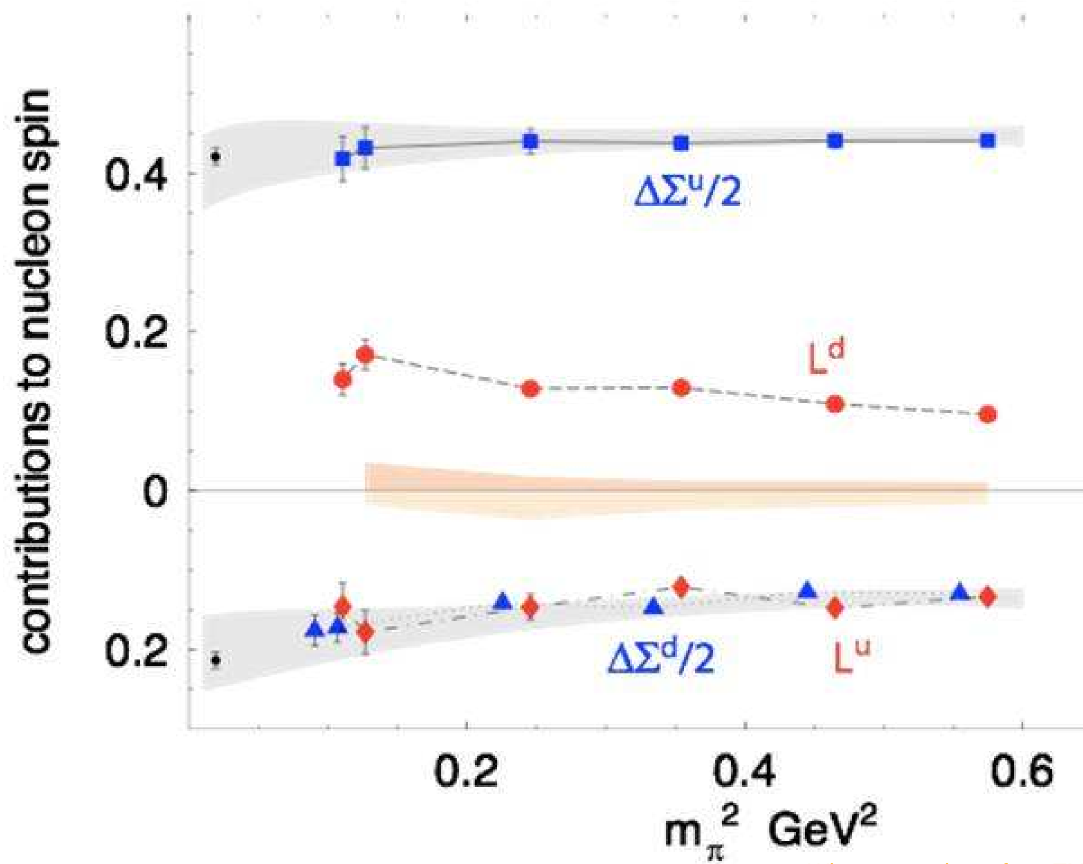
- lattice QCD $\Rightarrow$ moments of GPDs (LHPC; QCDSF)

$\hookrightarrow$ insert in Ji-relation

$$\langle J_q^i \rangle = S^i \int dx \left[H_q(x,0) + E_q(x,0) \right] x .$$

$\hookrightarrow$ $L_q^z = J_q^z - \frac{1}{2}\Delta q$

- L_u, L_d both large!
- present calcs. show $L_u + L_d \approx 0$, but
  - disconnected diagrams ..?
  - m_π^2 extrapolation
  - parton interpret. of L_q...



Angular Momentum in QCD (Jaffe & Manohar)

- define OAM on a light-like hypesurface rather than a space-like hypersurface

$$\tilde{J}^3 = \int d^2x_\perp \int dx^- M^{12+}$$

where $x^- = \frac{1}{\sqrt{2}}\left(x^0 - x^-\right)$ and $M^{12+} = \frac{1}{\sqrt{2}}\left(M^{120} + M^{123}\right)$

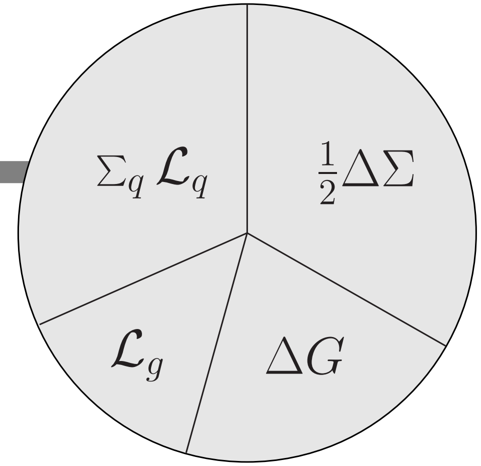
- Since $\partial_\mu M^{12\mu} = 0$

$$\int d^2\mathbf{x}_\perp \int dx^- M^{12+} = \int d^2\mathbf{x}_\perp \int dx^3 M^{120}$$

(compare electrodynamics: $\vec{\nabla} \cdot \vec{B} = 0 \quad \Rightarrow$ flux in = flux out)

- use eqs. of motion to get rid of ‘time’ (∂_+ derivatives) & integrate by parts whenever a total derivative appears in the T^{i+} part of M^{12+}

Jaffe/Manohar decomposition



- in light-cone framework & light-cone gauge $A^+ = 0$ one finds for $J^z = \int dx^- d^2\mathbf{r}_\perp M^{+xy}$

$$\frac{1}{2} = \frac{1}{2}\Delta\Sigma + \sum_q \mathcal{L}_q + \Delta G + \mathcal{L}_g$$

where ($\gamma^+ = \gamma^0 + \gamma^z$)

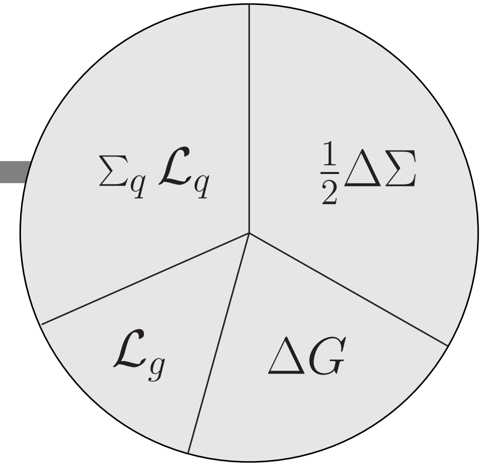
$$\mathcal{L}_q = \int d^3r \, \langle P,S| \, \bar{q}(\vec{r})\gamma^+ \left(\vec{r} \times i\vec{\partial}\right)^z q(\vec{r}) \, |P,S\rangle$$

$$\Delta G = \varepsilon^{+-ij} \int d^3r \, \langle P,S| \, \mathrm{Tr} F^{+i} A^j \, |P,S\rangle$$

$$\mathcal{L}_g = 2 \int d^3r \, \langle P,S| \, \mathrm{Tr} F^{+j} \left(\vec{x} \times i\vec{\partial}\right)^z A^j \, |P,S\rangle$$

Jaffe/Manohar decomposition

$$\frac{1}{2} = \frac{1}{2}\Delta\Sigma + \sum_q \mathcal{L}_q + \Delta G + \mathcal{L}_g$$



- $\Delta\Sigma = \sum_q \Delta q$ from polarized DIS (or lattice)
- ΔG from $\overrightarrow{p}\overleftarrow{p}$ or polarized DIS (evolution)

$\hookrightarrow$ ΔG gauge invariant, but local operator only in light-cone gauge

- $\int dx x^n \Delta G(x)$ for $n \geq 1$ *can* be described by manifestly gauge inv. local op. ($\longrightarrow$ lattice)
- $\mathcal{L}_q$, $\mathcal{L}_g$ independently defined, but
  - no exp. identified to access them
  - not accessible on lattice, since nonlocal except when $A^+ = 0$
- parton net OAM $\mathcal{L} = \mathcal{L}_g + \sum_q \mathcal{L}_q$ by subtr. $\mathcal{L} = \frac{1}{2} - \frac{1}{2}\Delta\Sigma - \Delta G$
- in general, $\mathcal{L}_q \neq L_q$ $\qquad$ $\mathcal{L}_g + \Delta G \neq J_g$
- makes no sense to ‘mix’ Ji and JM decompositions, e.g. $J_g - \Delta G$ has no fundamental connection to OAM

$L_q \neq \mathcal{L}_q$

- L_q matrix element of

$$q^\dagger \left[\vec{r} \times \left(i\vec{\partial} - g\vec{A}\right)\right]^z q = \bar{q}\gamma^0 \left[\vec{r} \times \left(i\vec{\partial} - g\vec{A}\right)\right]^z q$$

- $\mathcal{L}_q^z$ matrix element of $(\gamma^+ = \gamma^0 + \gamma^z)$

$$\bar{q}\gamma^+ \left[\vec{r} \times i\vec{\partial}\right]^z q\Big|_{A^+=0}$$

- For nucleon at rest, matrix element of L_q same as that of $\bar{q}\gamma^+ \left[\vec{r} \times \left(i\vec{\partial} - g\vec{A}\right)\right]^z q$

$\hookrightarrow$ even in light-cone gauge, L_q^z and $\mathcal{L}_q^z$ still differ by matrix element of $q^\dagger \left(\vec{r} \times g\vec{A}\right)^z q\Big|_{A^+=0} = q^\dagger \left(xgA^y - ygA^x\right) q\Big|_{A^+=0}$

Summary part 1:

- Ji: $J^z = \frac{1}{2}\Delta\Sigma + \sum_q L_q + J_g$
- Jaffe: $J^z = \frac{1}{2}\Delta\Sigma + \sum_q \mathcal{L}_q + \Delta G + \mathcal{L}_g$
- ΔG can be defined without reference to gauge (and hence gauge invariantly) as the quantity that enters the evolution equations and/or $\vec{p}\overleftarrow{p}$

$\hookrightarrow$ represented by simple (i.e. local) operator only in LC gauge and corresponds to the operator that one would naturally identify with 'spin' only in that gauge

- in general $L_q \neq \mathcal{L}_q$ or $J_g \neq \Delta G + \mathcal{L}_g$, but
- how significant is the difference between L_q and $\mathcal{L}_q$, etc. ?

OAM in scalar diquark model

[M.B. + Hikmat Budhathoki Chhetri (BC), PRD **79**, 071501 (2009)]

- toy model for nucleon where nucleon (mass M) splits into quark (mass m) and scalar 'diquark' (mass λ)

$\hookrightarrow$ light-cone wave function for quark-diquark Fock component

$$\psi^{\uparrow}_{+\frac{1}{2}}(x,\mathbf{k}_\perp) = \left(M + \frac{m}{x}\right)\phi \qquad \psi^{\uparrow}_{-\frac{1}{2}} = -\frac{k^1 + ik^2}{x}\phi$$

with $\phi = \frac{c/\sqrt{1-x}}{M^2 - \frac{\mathbf{k}_\perp^2 + m^2}{x} - \frac{\mathbf{k}_\perp^2 + \lambda^2}{1-x}}$.

- quark OAM according to JM: $\mathcal{L}_q = \int_0^1 dx \int \frac{d^2\mathbf{k}_\perp}{16\pi^3}(1-x)\left|\psi^{\uparrow}_{-\frac{1}{2}}\right|^2$
- quark OAM according to Ji: $L_q = \frac{1}{2}\int_0^1 dx\, x\left[q(x) + E(x,0,0)\right] - \frac{1}{2}\Delta q$

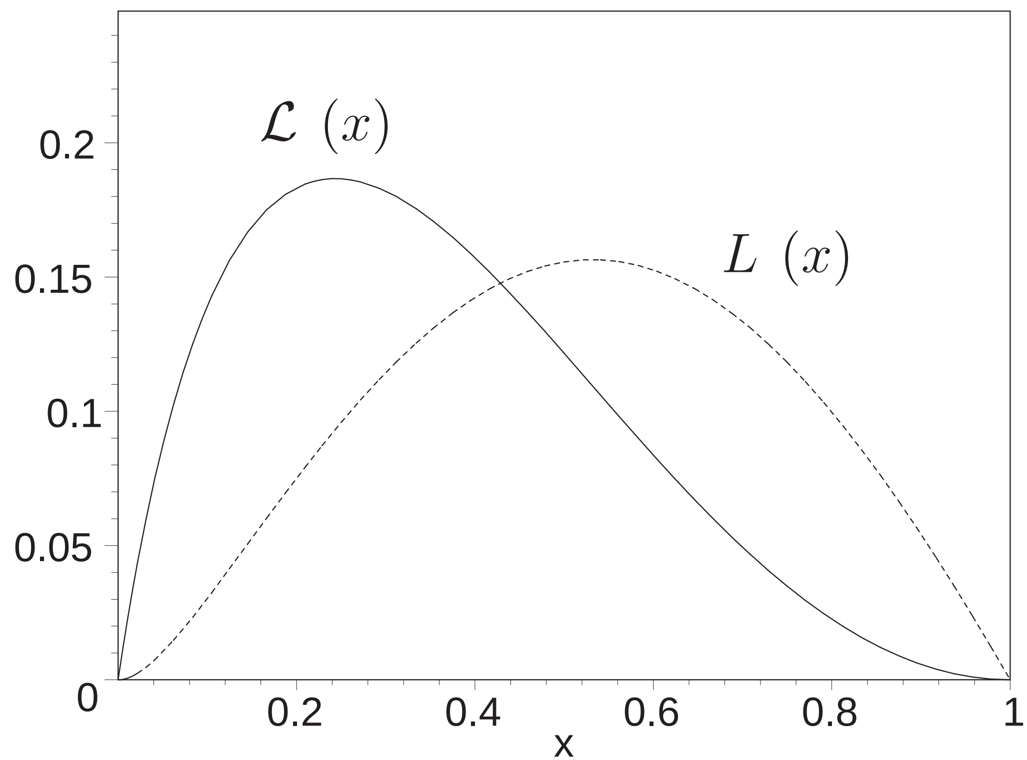
$\rightsquigarrow$ (using Lorentz inv. regularization, such as Pauli Villars subtraction) both give identical result, i.e. $L_q = \mathcal{L}_q$

- not surprising since scalar diquark model is <u>not</u> a gauge theory

OAM in scalar diquark model

- But, even though $L_q = \mathcal{L}_q$ in this non-gauge theory

$$\mathcal{L}_q(x) \equiv \int \frac{d^2\mathbf{k}_\perp}{16\pi^3}(1-x)\left|\psi^{\uparrow}_{-\frac{1}{2}}\right|^2 \neq \frac{1}{2}\left\{x\left[q(x)+E(x,0,0)\right]-\Delta q(x)\right\} \equiv L_q(x)$$



$\hookrightarrow$ ‘unintegrated Ji-relation’ does not yield x-distribution of OAM

OAM in QED

- light-cone wave function in $e\gamma$ Fock component

$$\Psi^{\uparrow}_{+\frac{1}{2}+1}(x,\mathbf{k}_\perp) = \sqrt{2}\frac{k^1 - ik^2}{x(1-x)}\phi \qquad \Psi^{\uparrow}_{+\frac{1}{2}-1}(x,\mathbf{k}_\perp) = -\sqrt{2}\frac{k^1 + ik^2}{1-x}$$

$$\Psi^{\uparrow}_{-\frac{1}{2}+1}(x,\mathbf{k}_\perp) = \sqrt{2}\left(\frac{m}{x} - m\right)\phi \qquad \Psi^{\uparrow}_{-\frac{1}{2}+1}(x,\mathbf{k}_\perp) = 0$$

- OAM of e^- according to Jaffe/Manohar

$$\mathcal{L}_e = \int_0^1 dx \int d^2\mathbf{k}_\perp \left[(1-x)\left|\Psi^{\uparrow}_{+\frac{1}{2}-1}(x,\mathbf{k}_\perp)\right|^2 - \left|\Psi^{\uparrow}_{+\frac{1}{2}+1}(x,\mathbf{k}_\perp)\right|^2\right]$$

- e^- OAM according to Ji $L_e = \frac{1}{2}\int_0^1 dx\, x\left[q(x) + E(x,0,0)\right] - \frac{1}{2}\Delta q$

$\rightsquigarrow$ $\mathcal{L}_e = L_e + \frac{\alpha}{4\pi} \neq L_e$

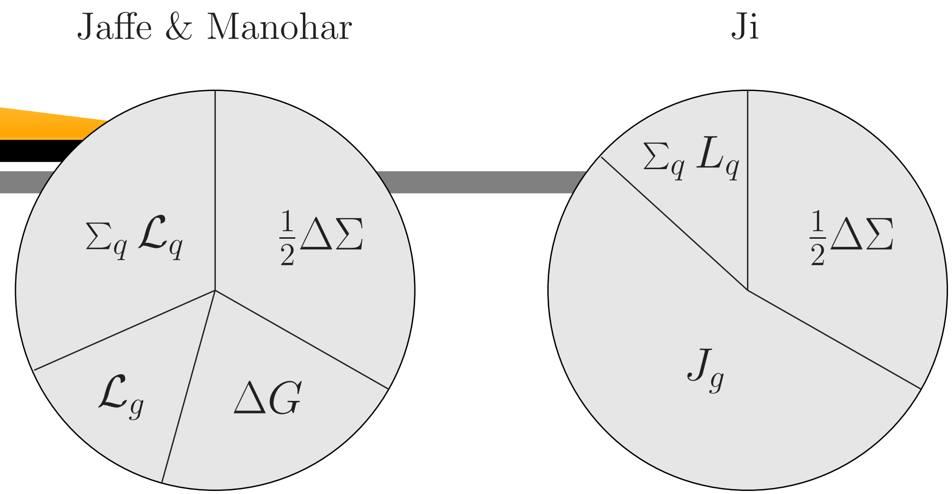
- Likewise, computing J_γ from photon GPD, and $\Delta\gamma$ and $\mathcal{L}_\gamma$ from light-cone wave functions and <u>defining</u> $\hat{L}_\gamma \equiv J_\gamma - \Delta\gamma$ yields $\hat{L}_\gamma = \mathcal{L}_\gamma + \frac{\alpha}{4\pi} \neq \mathcal{L}_\gamma$

- $\frac{\alpha}{4\pi}$ appears to be small, but here $\mathcal{L}_e$, L_e are all of $\mathcal{O}(\frac{\alpha}{\pi})$

OAM in QCD

- ↪ 1-loop QCD: $\mathcal{L}_q - L_q = \frac{\alpha_s}{3\pi}$
- recall (lattice QCD): $L_u \approx -.15$; $L_d \approx +.15$
- QCD evolution yields negative correction to L_u and positive correction to L_d
- ↪ evolution suggested (A.W.Thomas) to explain apparent discrepancy between quark models (low Q^2) and lattice results ($Q^2 \sim 4GeV^2$)
- above result suggests that $\mathcal{L}_u > L_u$ and $\mathcal{L}_d > L_d$
- additional contribution (with same sign) from vector potential due to spectators (MB, to be published)
- ↪ possible that lattice result consistent with $\mathcal{L}_u > \mathcal{L}_d$

Summary



- inclusive $\vec{e}\,\overleftarrow{p}$ / $\vec{p}\,\overleftarrow{p}$ provide access to
  - quark spin $\frac{1}{2}\Delta q$
  - gluon spin ΔG
  - parton grand total OAM $\mathcal{L} \equiv \mathcal{L}_g + \sum_q \mathcal{L}_q = \frac{1}{2} - \Delta G - \sum_q \Delta q$
- DVCS & polarized DIS and/or lattice provide access to
  - quark spin $\frac{1}{2}\Delta q$
  - J_q & $L_q = J_q - \frac{1}{2}\Delta q$
  - $J_g = \frac{1}{2} - \sum_q J_q$
- $J_g - \Delta G$ does <u>not</u> yield gluon OAM $\mathcal{L}_g$
- $L_q - \mathcal{L}_q = \mathcal{O}(0.1 * \alpha_s)$ for O (α_s) dressed quark

Announcement:

- workshop on Orbital Angular Momentum of Partons in Hadrons
- ECT* 9-13 November 2009
- organizers: M.B. & Gunar Schnell
- confirmed participants: M.Anselmino, H.Avakian, A.Bacchetta, L.Bland, D.Boer, D.Fields, L.Gamberg, G.Goldstein, M.Grosse-Perdekamp, P.Hägler, X.Ji, R.Kaiser, E.Leader, N.Makins, A.Miller, D.Müller, P.Mulders, A.Schäfer, G.Schierholz, O.Teryaev, W.Vogelsang, F.Yuan

Summary

- distribution of $\perp$ polarized quarks in unpol. target described by chirally odd GPD $\bar{E}_T^q = 2\bar{H}_T^q + E_T^q$

$\hookrightarrow$ attractive FSI $\Rightarrow$ measurement of $h_1^\perp$ (DY,SIDIS) provides information on $\bar{E}_T^q$ and hence on spin-orbit correlations

- expect:

$$h_1^{\perp,q} < 0 \qquad\qquad |h_1^{\perp,q}| > |f_{1T}^q|$$

- x^2-moment of chirally odd twist-3 PDF $e(x)$ $\longrightarrow$ **transverse force on transversly polarized quark in unpolarized target** ($\longrightarrow$ Boer-Mulders)
- see also: M.B., A.Miller, and W.-D.Nowak, 'Spin-Polarized High-Energy Scattering of Charged Leptons on Nucleons', hep-ph/0812.2208

Impact parameter dependent PDFs

- No relativistic corrections (Galilean subgroup!)

$\hookrightarrow$ corrolary: interpretation of 2d-FT of $F_1(Q^2)$ as charge density in transverse plane also free from relativistic corrections

- $q(x, \mathbf{b}_\perp)$ has probabilistic interpretation as number density ($\Delta q(x, \mathbf{b}_\perp)$ as difference of number densities)

- Reference point for IPDs is transverse center of (longitudinal) momentum $\mathbf{R}_\perp \equiv \sum_i x_i \mathbf{r}_{i,\perp}$

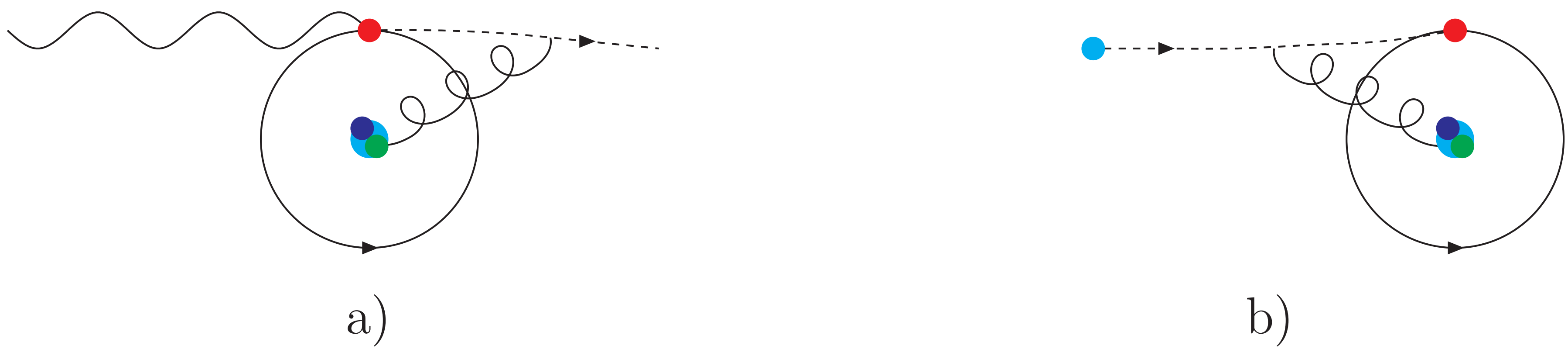
$\hookrightarrow$ for $x \to 1$, active quark 'becomes' COM, and $q(x, \mathbf{b}_\perp)$ must become very narrow (δ-function like)

$\hookrightarrow$ $H(x, 0, -\mathbf{\Delta}_\perp^2)$ must become $\mathbf{\Delta}_\perp$ indep. as $x \to 1$ (MB, 2000)

$\hookrightarrow$ consistent with lattice results for first few moments

- Note that this does not necessarily imply that 'hadron size' goes to zero as $x \to 1$, as separation $\mathbf{r}_\perp$ between active quark and COM of spectators is related to impact parameter $\mathbf{b}_\perp$ via $\mathbf{r}_\perp = \frac{1}{1-x}\mathbf{b}_\perp$.

$f_{1T}^{\perp}(x, \mathbf{k}_\perp)_{DY} = -f_{1T}^{\perp}(x, \mathbf{k}_\perp)_{SIDIS}$



- time reversal: FSI $\leftrightarrow$ ISI

SIDIS: compare FSI for 'red' q that is being knocked out with ISI for an anti-red $\bar{q}$ that is about to annihilate that bound q

$\hookrightarrow$ FSI for knocked out q is attractive

DY: nucleon is color singlet $\rightarrow$ when to-be-annihilated q is 'red', the spectators must be anti-red

$\hookrightarrow$ ISI with spectators is repulsive

What is a Polarizability?

- Polarizability is the relative tendency of a charge distribution, like the electron cloud of an atom or molecule, to be distorted from its normal shape by an external electric field, which may be caused by the presence of a nearby ion or dipole (Wikipedia)
- It may be consistent with this original use of the term to enlarge the definition to encompass all observables that describe the ease with which a system can be distorted in response to an applied field or force
- Suppose one enlarges this definition to encompass ‘how the color electric and magnetic field responds to the spin of the nucleon’

$\hookrightarrow$ many other obeservables also become ‘polarizabilities’, e.g.

  - Δq, as is describes how the quark spin responds to the spin of the nucleon
  - $\vec{\mu}_N$, as it describes how the magnetic field of the nucleon responds to the spin of the nucleon
  - $\vec{L}_q$, as it describes how the quark orbital angular momentum responds to the spin of the nucleon
  - as well as many other ‘static’ properties of the nucleon

Sivers Mechanism in $A^+ = 0$ gauge

- Gauge link along light-cone trivial in light-cone gauge

$$U_{[0,\infty]} = P \exp \left(ig \int_0^\infty d\eta^- A^+(\eta) \right) = 1$$

$\hookrightarrow$ Puzzle: Sivers asymmetry seems to vanish in LC gauge (time-reversal invariance)!

- X.Ji: fully gauge invariant definition for $P(x, \mathbf{k}_\perp)$ requires additional gauge link at $x^- = \infty$

$$\begin{aligned} f(x, \mathbf{k}_\perp) &= \int \frac{dy^- d^2\mathbf{y}_\perp}{16\pi^3} e^{-ixp^+y^- + i\mathbf{k}_\perp \cdot \mathbf{y}_\perp} \\ &\times \left\langle p, s \left| \bar{q}(y) \gamma^+ U_{[y^-, \mathbf{y}_\perp; \infty^-, \mathbf{y}_\perp]} U_{[\infty^-, \mathbf{y}_\perp, \infty^-, \mathbf{0}_\perp]} U_{[\infty^-, \mathbf{0}_\perp; 0^-, \mathbf{0}_\perp]} q(0) \right| p, s \right\rangle \end{aligned}$$

back